UNIT III

TRANSFORMERS

OUTPUT EQUATIONS

1. OUTPUT EQUATION OF SINGLE PHASE TRANSFORMER

The equation which relates the rated kVA output of a transformer to the area of core and window is called output equation. In transformers the output kVA depends on flux density and ampere-turns. The flux density is related to core area and the ampere-turns is related to window area.

The simplified cross-section of core type and shell type single phase transformers are shown in figures (1) and (2). The low voltage winding is placed nearer to the core in order to reduce the insulation requirement. The space inside the core is called window and it is the space available for accommodating the primary and secondary winding. The window area is shared between the winding and their insulations.

![Cross-section of core type single phase transformer](image1)

**Fig 1: Cross-section of core type single phase transformer**

![Cross-section of shell type single phase transformer](image2)

**Fig 2: Cross-section of shell type single phase transformer**

The induced emf in a transformer, \( E = 4.44 f \Phi_m \) T Volts

Emf per turn, \( E_t = E / T = 4.4 f \Phi_m \) Volts \hspace{1cm} (1)

The window in single phase transformer contains one primary and one secondary winding. The window space factor \( K_w \) is the ratio of conductor area in window to total area of window.

Window space factor, \( K_w = \frac{\text{Conductor area in window}}{\text{Total area of window}} = \frac{A_c}{A_w} \)

\( \therefore \) Conductor area in window, \( A_c = K_w A_w \) \hspace{1cm} (2)
The current density $\delta$ is same in both the windings.

\[ \therefore \text{Current density, } \delta = \frac{I_p}{a_p} = \frac{I_s}{a_s} \]

Area of cross-section of primary conductor, \( a_p = \frac{I_p}{\delta} \)

Area of cross-section of secondary conductor, \( a_s = \frac{I_s}{\delta} \)

If we neglect magnetizing mmf then primary ampere turns is equal to secondary ampere turns.

\[ \therefore \text{Ampere turns, } AT = I_pT_p = I_sT_s \]

Total copper area in window \( A_c = \text{Copper area of primary winding} + \text{Copper area of secondary winding} \)

\[ = \text{Number of primary turns} \times \text{area of cross-section of primary conductor} + \text{Number of secondary turns} \times \text{area of cross-section of secondary conductor} \]

\[ = \frac{1}{\delta}(T_pI_p + T_sI_s) = \frac{1}{\delta}(AT + AT) \]

\[ = \frac{2AT}{\delta} \]

Equating equation (2) and (4), we get

\[ K_wA_w = 2 \frac{AT}{\delta} \]

\[ \therefore \text{Ampere turns, } AT = \frac{1}{2}K_wA_w \delta \]

The kVA rating of single phase transformer is given by,

\[ \text{kVA rating, } Q = V_pI_p \times 10^{-3} \approx E_pI_p \times 10^{-3} \quad \left( \because E_p \approx V_p \right) \]

\[ = \frac{E_p}{T_p} T_pI_p \times 10^{-3} \quad \left( \because E_t = \frac{E_p}{T_p} \text{ and } AT = T_pI_p \right) \]

\[ = E_t \times AT \times 10^{-3} \]

Substituting equations (1) and (4) in equation (5), we get

\[ Q = 4.44 \phi_m \frac{K_wA_w \delta}{2} \times 10^{-3} \]
\[ Q = 2.22 \times 10^{-3} f \frac{B_m}{A_i} \]

The equation (6) is the output equation of single phase transformer.

2. OUTPUT EQUATION OF THREE PHASE TRANSFORMER

The simplified cross-section of core type three phase transformer is shown in fig 3. The cross-section has three limbs and two windows. Each limb carries the low voltage and high voltage winding of a phase.

In case of three phase transformer, each window has two primary and two secondary windings.

Hence the area of copper is taken twice that of single phase core type transformer.

The window space factor \( K_w \) is the ratio of conductor area in window to total area of window,

\[ K_w = \frac{\text{Conductor area in window}}{\text{Total area of window}} = \frac{A_c}{A_w} \]

\[ A_c = K_w A_w \quad \text{--- (8)} \]

The current density \( \delta \) is same in both the windings.

\[ \delta = \frac{I_p}{a_p} = \frac{I_s}{a_s} \]

Area of cross-section of primary conductor, \( a_p = \frac{I_p}{\delta} \)

Area of cross-section of secondary conductor, \( a_s = \frac{I_s}{\delta} \)
If we neglect magnetizing mmf then primary ampere turns is equal to secondary ampere turns.

\[ \therefore \text{Ampere turns, } AT = I_p T_p = I_s T_s \]

\[
A_c = \left\{ \begin{array}{c}
2 \times \text{Number of primary turns} \times \text{area of cross-section of primary conductor} \\
2 \times \text{Number of secondary turns} \times \text{area of cross-section of secondary conductor}
\end{array} \right.
\]

\[ = 2 T_p a_p + 2 T_s a_s \]

\[ = 2 T_p \frac{I_p}{\delta} + 2 T_s \frac{I_s}{\delta} \]

\[ = \frac{2}{\delta} (T_p I_p + T_s I_s) \]

\[ = \frac{2}{\delta} (AT + AT) \]

\[ = \frac{4 AT}{\delta} \]

Equating equation (8) and (9), we get

\[ \frac{4 AT}{\delta} = K_w A_w \]

\[ \therefore \text{Ampere-turn, } AT = \frac{K_w A_w \delta}{4} \]

The equation (12) is the output equation of three phase transformer.

The equations (6) and (12) hold good for core and shell type transformers.
OUTPUT EQUATION – VOLT PER TURN

The transformer design starts with selection of an appropriate value for emf per turn. Hence an equation for emf per turn can be developed by relating output kVA, magnetic and electric loading. In transformers the ratio of specific magnetic and electric loading is specified rather than actual value of specific loadings.

Let,

\[ r = \frac{\phi_m}{AT} \]

The volt-ampere per phase of a transformer is given by the product of voltage and current per phase. Considering the primary voltage and current per phase we can write,

\[
kVA \text{ per phase, } Q = V_p I_p \times 10^{-3} = 4.44 f \phi_m T_p I_p \times 10^{-3} = 4.44 f \phi_m AT \times 10^{-3} = 4.44 f \phi_m \frac{\phi_m}{r} \times 10^{-3}
\]

\[ \therefore \phi_m^2 = \frac{Q r}{4.44 f \times 10^{-3}} \]

\[ \phi_m = \sqrt{\frac{Q r \times 10^3}{4.44 f}} \]

We know that,

\[ \text{Emf per turn, } E_t = 4.44 f \phi_m \quad \text{----------------- (13)} \]

Substituting equation (13) in equation (14) we get,

\[ E_t = 4.44 f \sqrt{\frac{Q r \times 10^3}{4.44 f}} = \sqrt{4.44 f r \times 10^3} \sqrt{Q} = K \sqrt{Q} \]

\[ E_t = K \sqrt{Q} \quad \text{----------------- (15)} \]

where, \( K = \sqrt{4.44 f r \times 10^3} = \sqrt{4.44 f \times \frac{\phi_m}{AT} \times 10^3} \)

From equation (15) we can say that the emf per turn is directly proportional to \( K \). The value of \( K \) depends on the type, service condition and method of construction of transformer. The value of \( K \) for different types of transformers are listed in table.
### Transformer Table

<table>
<thead>
<tr>
<th>Transformer Type</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single phase shell type</td>
<td>1.0 to 1.2</td>
</tr>
<tr>
<td>Single phase core type</td>
<td>0.75 to 0.85</td>
</tr>
<tr>
<td>Three phase shell type</td>
<td>1.3</td>
</tr>
<tr>
<td>Three phase core type, distribution transformer</td>
<td>0.45</td>
</tr>
<tr>
<td>Three phase core type, power transformer</td>
<td>0.6 to 0.7</td>
</tr>
</tbody>
</table>

### Ratio of Iron Loss to Copper Loss

Ratio of iron loss to copper loss

\[
\frac{P_i}{P_c} = \frac{p_i G_i}{p_c G_c}
\]

Where

- \(G_i\) = weight of active iron, kg
- \(G_c\) = weight of copper, kg
- \(p_i\) = loss in iron per kg, W
- \(p_c\) = loss in copper per kg, W

The ratio of weight of iron to weight of copper generally lies between 1.5 to 3.0 for distribution transformers.

### Relation Between Core Area and Weight of Iron and Copper

Area of core

\[
A_i = \sqrt{\frac{Q G_i}{f B_m G_c}}
\]

### Optimum Design

Transformers may be designed to make one of the following quantities as minimum.

(i) Total volume
(ii) Total weight
(iii) Total cost
(iv) Total losses

In general, these requirements are contradictory and it is normally possible to satisfy only one of them. All these quantities vary with ratio \(r = \Phi_m / AT\). If we choose a high value of \(r\), the flux becomes larger and consequently a large core cross-section is needed which results in higher volume, weight and cost of iron and also gives a higher iron loss. On the other hand, owing to decrease in the value of \(AT\) the volume, weight and cost of copper required decrease and also the \(FR\) losses decrease. Thus we conclude that the value of \(r\) is a controlling factor for the above mentioned quantities.

### 1. Design for Minimum Cost

Let us consider a single phase transformer. In kVA output is:

\[
Q = 2.22 f B_m \delta K_w A_w A_i \times 10^{-3} = 2.22 f B_m \delta A_r A_i \times 10^{-3}
\]
Where $A_e = k_e A_w$

Assuming that the flux and current densities are constant, we see that for a transformer of given rating the product $A_e A_i$ is constant.

Let this product $A_e A_i = M^2$ \hspace{1cm} ...(i)

The optimum design problem is, therefore, that of determining the minimum value of total cost.

Now,
\[
\rho = \frac{\Phi_n}{AT} \quad \text{and} \quad \phi_m = B_m A_i \quad \text{and} \quad AT = \delta K_n A_n / 2 = \delta A / 2
\]

\[
\therefore \quad \rho = \frac{2B_m A_i}{\delta A} \quad \text{or} \quad \frac{A_i}{A_e} = \frac{\delta}{2B_m} \quad \rho = \beta
\] \hspace{1cm} ...(ii)

where $\beta$ is a function of $\rho$ only as $B_m$ and $\delta$ are constant.

Thus from (i) and (ii) we have

\[
A_i = M \sqrt{\beta} \quad \text{and} \quad A_e = M / \sqrt{\beta}
\]

Let

$C_i = \text{total cost of transformer active materials,}$

$C_e = \text{total cost of iron, and } C_c = \text{total cost of conductor}$

\[
C_i = C_i + C_c = c_i g_i l_i A_i + c_c g_c L_{mt} A_c
\]

$c_i$ and $c_c$ are the specific costs of iron and copper respectively.

Now,
\[
C_i = c_i g_i l_i M \sqrt{\beta} + c_c g_c L_{mt} M / \sqrt{\beta}
\]

where $g_i =$ weight per m$^3$ of iron, kg ; $g_c =$ weight per m$^3$ of copper, kg ;

Differentiating $C_i$ with respect to $\beta$,
\[
\frac{dC_i}{d\beta} = \frac{1}{2} c_i g_i l_i M (\beta)^{-1/2} - \frac{1}{2} c_c g_c L_{mt} M \beta^{-3/2}
\]

For minimum cost $\frac{dC_i}{d\beta} = 0$

\[
\therefore \quad c_i g_i l_i = c_c g_c L_{mt} \beta^{-1} \quad \text{or} \quad c_i g_i l_i = c_c g_c L_{mt} \frac{A_c}{A_i}
\]

or
\[
\frac{c_i g_i l_i}{A_i} = c_c g_c L_{mt} A_c \quad \text{or} \quad c_i G_i = c_c G_c
\]

or
\[
C_i = C_c
\]

Hence, for minimum total cost, the cost of iron must equal the cost of conductor.

Now
\[
G_i / G_c = c_i / c_c \quad \text{for minimum cost.}
\]

Knowing the value of specific costs of iron and conductor the ratio of weight of iron to conductor can be determined.

Similar conditions apply to other quantities e.g.,

For minimum volume of transformer, Volume of iron = volume of conductor
\[
\therefore \quad G_i / g_i = G_c / g_c \quad i.e., \quad G_i / G_c = g_i / g_c
\]
For minimum weight of transformer
weight of iron = weight of conductor  \( i.e. \),  \( G_i = G_c \).

For minimum losses in transformer \( i.e. \), for maximum efficiency,
iron loss = \( P_R \) loss in conductor  \( i.e. \),  \( P_i = x^2 P_e \).

2. DESIGN FOR MINIMUM LOSS OR MAXIMUM EFFICIENCY

Total losses at full load = \( P_i + P_c \)

At any fraction \( x \) of full load, the total losses are  \( P_i + x^2 P_c \)

If \( Q \) is the output at full load, the output at fraction \( x \) of full load is \( xQ \).

\[ \therefore \text{Efficiency at output } xQ, \eta_x = \frac{xQ}{xQ + P_i + x^2 P_c} \]

This efficiency is maximum when \( \frac{d\eta_x}{dx} = 0 \)

Differentiating \( \eta_x \) we have

\[ \frac{d\eta_x}{dx} = \frac{(xQ + P_i + x^2 P_c) Q - xQ(Q + 2xP_c)}{(xQ + P_i + x^2 P_c)^2} \]

For maximum efficiency, \( (xQ + P_i + x^2 P_c) Q - xQ(Q + 2xP_c) = 0 \)

we have:

\[ \frac{P_i}{P_c} = \frac{P_i}{P_c} \frac{G_i}{G_c} \]

\[ \therefore \quad x^2 = \frac{P_i}{P_c} \frac{G_i}{G_c} \quad \text{or} \quad \frac{G_i}{G_c} = x^2 \frac{P_c}{P_i} \text{ for maximum efficiency.} \]

Now knowing the values of densities in iron and copper the specific losses \( p_i \) and \( p_c \) can be determined and the value of \( x \) \( i.e. \), the fraction of full load where the maximum efficiency occurs depends upon the service conditions of the transformer and is, therefore, known.

WINDOW SPACE FACTOR

The window space factor is defined as the ratio of copper area in the window to total window area.

\[ K_w = \frac{\text{Conductor area in window}}{\text{total area of window}} = \frac{A_c}{A_w} \]

It depends upon the relative amounts of insulation and copper provided, which in turn depends upon the voltage rating and output of transformers. The following formulae may be used for estimating the value of window space factor.

\[ K_w = \frac{8}{30 + kV} \quad \text{for } 20 \text{ kVA rating} \]

\[ K_w = \frac{10}{30 + kV} \quad \text{for } 50 \text{ to } 200 \text{ kVA rating} \]

\[ K_w = \frac{12}{30 + kV} \quad \text{for } 1000 \text{ kVA rating} \]

Where \( kV \) is the voltage of h.v. winding in kilo-volt.
DESIGN OF CORE

- The core section of core type of transformers may be rectangular, square or stepped.
- Shell type transformers use cores with rectangular cross section.

The fig. shows the cross section of transformer cores.

- With small size transformers, rectangular core can be used with either circular or rectangular coils.
- With medium size transformers, square core can be used.
- With large transformers, cruciform (stepped) cores, which utilize the space better are used.
- The circle represents the inner surface of the tubular form carrying the windings. This circle is known as the circumscribing circle.
- Circular coils are preferred over rectangular coils because of their superior mechanical characteristics. On circular coils the forces are radial and there is no tendency for the coil to change its shape. On rectangular coils the forces are perpendicular to the conductors and tend to give the coil a circular form, thus deforming it. Hence circular coils are employed in high voltage and high capacity transformers.

Note:

\[ S_t = k_i = \text{Stacking factor} \]

Stacking factor, \( S_t = \frac{\text{Area of cross-section of iron in the core}}{\text{Area of cross-section of the core including the insulation area}} \]

The usual value of stacking factor is 0.9.

By increasing the number of steps, the area of circumscribing circle is more effectively utilized. The most economical dimensions of various steps for a multi-stepped core can be calculated. The results are tabulated in table.
SELECTION OF CORE AREA AND TYPE OF CORE

Stepped core cross-section is preferred to obtain the optimum core area within the circumscribing circle of the core. The core area is determined by the number of steps, grade of steel, insulation or laminations and type of clamping. With increase of number of steps core area increases but cost increases. For larger rated transformers high tensile strength clamps are used for clamping core lamination and it provides increased core area for any fixed core diameter. In order to keep the hot spot temperature within specified limits, sufficient number of ducts are to be provided.

CALCULATION OF CORE AREA

The voltage per turn is calculated from Eqn.

\[ E_t = K \sqrt{Q} \]

where, \( K = \sqrt{\frac{4.44 f \times \Phi_m \times 10^3}{AT}} \)

Now, flux \( \Phi_m = \frac{E_t}{4.44 f} \)

Therefore, the value of flux in the core can be calculated. The area of the core is found out by assuming a suitable value of maximum flux density \( B_m \).

Net core area required \( A_t = \frac{\Phi_m}{B_m} \)

and gross core area \( A_{gt} = \frac{A_t}{k_i} \)

DESIGN OF WINDINGS

Number of turns in primary winding \( T_p = \frac{V_p}{E_t} \)

Number of turns in secondary winding \( T = \frac{V}{E_t} \)

Current in primary winding \( I_p = \frac{KVA \times 10^3}{V_p} \)
Area of each primary winding \( a_p = \frac{I_p}{\delta_p} \)

Area of each secondary winding \( a_s = \frac{I_s}{\delta_s} \)

- The current densities in the two windings should be taken equal in order to have minimum copper losses. i.e. \( \delta_p = \delta_s \)

- In practice, however, the current density in the relatively better cooled outer winding is made 5 percent greater than the inner winding.

**Position of winding relative to the core**

The l.v. winding is placed on the inner side nearer to the core with h.v. winding on the outside. This arrangement is used because the potential difference between l.v. winding and core (which is at earth potential) is small, there is less likelihood of a fault occurring between the two. Also, with l.v. winding placed nearer to the core, the insulation used between core and winding has a small thickness. However, in case h.v. winding is placed around the core, the insulation between the two has to be thick and this makes the length of mean turn large. This is clear from Fig. The cost of insulation is also higher with h.v. on the under side as same amount of insulation has to be used between h.v. and l.v. as is used between h.v. and core while if l.v. is on the inner side, the major insulation is only between l.v. and core.

![Diagram](image)

**Fig.** Position of windings relative to core.

The tappings are provided on the h.v. winding, therefore, it is very convenient to tap the winding as it is on the outside.
EXAMPLE: 01

Show that the output of a 3 phase core type transformer is:

\[ Q = 5.23 f B_m H d^2 H_w \times 10^{-3} \text{kVA}. \]

where \( f \) = frequency, Hz; \( B_m \) = maximum flux density, Wb/m²; \( d \) = effective diameter of core, m; \( H \) = magnetic potential gradient in limb, A/m; \( H_w \) = height of limb (window), m.

Solution

\[
\text{kVA output of a three phase transformer } Q = 3EI \times 10^{-3} \\
= 3 \times 4.44 f \Phi_m TI \times 10^{-3} = 3 \times 4.44 f B_m A_i TI \times 10^{-3}.
\]

In a three phase core type transformer each limb has one primary and one secondary winding wound on it and therefore total mmf over one limb = 2TI.

\[ \therefore \text{Magnetic potential gradient} \ H = \frac{\text{mmf}}{\text{height of limb}} = \frac{2TI}{H_w} \]

or \[ TI = \frac{HH_w}{2} \]

Also \[ A_i = (\pi/4)d^2 \]

Substituting the value of \( A_i \) and \( TI \) in the expression for \( Q \), we have

\[ Q = 3 \times 4.44 f B_m \times \frac{\pi}{4} d^2 \times H \frac{H_w}{2} \times 10^{-3} \]

\[ Q = 5.23 f B_m H d^2 H_w \times 10^{-3} \text{kVA}. \]

EXAMPLE: 02

Calculate the core and window areas required for a 1000 kVA, 6600/400 V, 50Hz, single phase core type transformer. Assume a maximum flux density of 1.25 Wb/m² and a current density of 2.5 A/mm². Voltage per turn = 30 V. Window space factor = 0.32.

Given data

\[
\begin{align*}
\text{kVA} &= 1000 & f &= 50 \text{ Hz} & B_m &= 1.25 \text{ Wb/m}^2 \\
V_p &= 6600 \text{ V} & V_s &= 400 \text{ V} & \delta &= 2.5 \text{ A/mm}^2 \\
E_i &= 30 \text{ V} & K_w &= 0.32 & \text{1-phase Core type}
\end{align*}
\]

Solution

Emf per turn, \( E_i = 4.44 f \Phi_m \)

\[ \therefore \Phi_m = \frac{E_i}{4.44 f} = \frac{30}{4.44 \times 50} = 0.1351 \text{ Wb} \]

Flux density, \( B_m = \frac{\Phi_m}{A_i} \)
The net area of cross-section of core

\[ A_i = \frac{\Phi_m}{B_m} = \frac{0.1351}{1.25} = 0.108 \text{ m}^2 = 0.108 \times 10^6 \text{ mm}^2 \]

\[ A_i = 0.108 \times 10^6 \text{ mm}^2 \]

The kVA rating of transformer, \( Q = 2.22 f B_m A_i K_w A_w \delta \times 10^3 \)

\[ \text{Window area, } A_w = \frac{Q}{2.22 f B_m A_i K_w \delta \times 10^{-3}} \]

\[
\begin{align*}
&= \frac{1000}{2.22 \times 50 \times 1.25 \times 0.108 \times 0.32 \times 2.5 \times 10^6 \times 10^{-3}} \\
&= 0.0834 \text{ m}^2 \\
\end{align*}
\]

\[ A_w = 0.0834 \times 10^6 \text{ mm}^2 \]

**EXAMPLE: 03**

The ratio of flux to full load mmf in a 400 kVA, 50 Hz, single phase core type power transformer is \(2.4 \times 10^{-6}\). Calculate the net iron area and the window area of the transformer. Maximum flux density in the core is 1.3 Wb/m^2, current density 2.7 A/mm^2 and window space factor 0.26. Also calculate the full load mmf.

**Given data**

- \( Q = 400 \text{ KVA} \)
- \( f = 50 \text{ Hz} \)
- \( \Phi_m / AT = 2.4 \times 10^{-6} \)
- \( B_m = 1.3 \text{ Wb/m}^2 \)
- \( \delta = 2.7 \text{ A/mm}^2 \)
- \( K_w = 0.26 \text{ single phase core type} \)

**Solution**

\[ K = \sqrt[4]{4.44 f (\Phi_m / AT) 10^3} = \sqrt[4]{4.44 \times 50 \times 24 \times 10^{-6} \times 10^3} = 0.732 \]

Voltage per turn \( E_t = K \sqrt{Q} = 0.732 \sqrt{400} = 14.64 \text{ V} \)

\[ \Phi_m = \frac{E_t}{4.44 f} = \frac{14.64}{4.44 \times 50} = 0.066 \text{ Wb} \]

Net iron area \( A_i = \frac{\Phi_m}{B_m} = \frac{0.066}{13} = 0.00507 \text{ m}^2 \).

\[ A_i = 0.00507 \text{ m}^2 \]

Window area of single phase transformer

\[ A_w = \frac{Q}{2.22 f B_m K_w \delta A_i \times 10^{-3}} \]
\[ A_w = \frac{400}{2.22 \times 50 \times 1.3 \times 0.26 \times 2.7 \times 10^8 \times 0.0507 \times 10^{-3}} = 0.0777 \text{ m}^2. \]

\[ AT = \frac{\Phi_m}{2.4 \times 10^{-5}} = \frac{0.066}{2.4 \times 10^{-5}} = 27500 \text{ A}. \]

**WINDOW DIMENSIONS**

- \( a \) = width of largest stamping,
- \( d \) = diameter of circumscribing circle,
- \( D \) = distance between centres of adjacent limbs,
- \( W_w \) = width of window,
- \( H_w \) = Height of window,
- \( = \) length of limb.

Fig. Transformer frame.

The leakage reactance is affected by the distance between adjacent limbs. If this distance is relatively small, the width of the winding is limited and this must be counter balanced by increasing the height of the winding. Thus the windings are long and thin. This arrangement leads to a low value of leakage reactance. If the height of the window is limited, the width of the window has to be increased in order to accommodate the coils. This results in short and wide coils giving a large value of leakage reactance.

The height and width of the window can be adjusted to give a suitable arrangement of windings and also to give a desired value of leakage reactance.

The area of the window depends upon total conductor area and the window space factor.

\[
\text{Area of window } A_w = \frac{\text{total conductor area}}{\text{window space factor}}
\]

\[
= \frac{2 \ a_p \ T_p}{K_w} \text{ for single phase transformers}
\]

\[
= \frac{4 \ a_p \ T_p}{K_w} \text{ for three phase transformers}
\]

Area of window \( A_w = \) height of window \( \times \) width of window = \( H_w \times W_w \)

The ratio of height to width of window, \( H_w/W_w \) is between 2 to 4

Assuming a suitable value for ratio \( H_w/W_w \), the height and width of window can be calculated

The width of window which gives the maximum output is \( W_w = D - d = 0.7 \ d \)
DESIGN OF YOKE

The entire core is divided into two portions. The vertical portion is called core or limb or leg. The horizontal portion is called yoke.

For rectangular section yokes,

\[
\text{Area of yoke } A_Y = \text{depth of yoke} \times \text{height of yoke} = D_Y \times H_Y
\]

where \( D_Y = \text{width of largest core stamping} = a \).

\( A_Y = (1.15 \text{ to } 1.25) A_{gi} \) for transformers using hot rolled steel

\( = A_{gi} \) for transformers using grain oriented steel.
When dealing with overall dimensions in transformer problems, refer to the following details and diagrams:

- \( a \) = width of largest stamping,
- \( d \) = diameter of circumscribing circle,
- \( D \) = distance between centres of adjacent limbs,
- \( W_w \) = width of window,
- \( H_w \) = Height of window,
- \( l \) = length of limb,
- \( H_y \) = height of yoke,
- \( H \) = Overall height of transformer over yokes or overall height of frame,
- \( W \) = length of yoke = overall length of frame.

We have the following relations for single phase core type transformers

\[
D = d + W_w, \quad D_y = a, \\
H = H_w + 2H_y, \quad W = D + a,
\]

Width over one limb = outer diameter of h.v. winding.
Width over two limbs = \( D \) + outer diameter of h.v. winding

---

Fig. Single phase shell type transformer.
We have, for a 3 phase core type transformers

\[ D = d + W_w; D_Y = a; H = H_w + 2H_Y; W = 2D + a. \]

Width over one limb = outer diameter of h.v. winding
Width over 3 limbs = 2D + outer diameter of h.v. winding.

For single phase shell type referring to Fig.

\[ D_Y = b, H_Y = a, W = 2W_w + 4a, H = H_w + 2a. \]

**EXAMPLE: 01**

*Show that the output of a 3 phase core type transformer is:*

\[ Q = 5.23 f B_m H_d^2 H_w \times 10^{-3} \text{kVA} \]

where \( f = \text{frequency, Hz} \); \( B_m = \text{maximum flux density, Wb/m}^2 \); \( d = \text{effective diameter of core, m} \);
\( H = \text{magnetic potential gradient in limb, A/m} \); \( H_w = \text{height of limb (window), m} \).

**Solution**

kVA output of a three phase transformer \( Q = 3EI \times 10^{-3} \)

In a three phase core type transformer each limb has one primary and one secondary winding wound on it and therefore total mmf over one limb = 2TI.

\[ \therefore \text{Magnetic potential gradient } H = \frac{\text{mmf}}{\text{height of limb}} \]

\[ = \frac{2TI}{H_w} \quad \text{or TI} = \frac{HH_u}{2} \quad \text{Also } A_i = (\pi/4)d^2 \]

Substituting the value of \( A_i \) and \( TI \) in the expression for \( Q \), we have

\[ Q = 3 \times 4.44 f B_m \times \frac{\pi}{4} d^2 \times H \frac{H_u}{2} \times 10^{-3} \]

\[ Q = 5.23 f B_m H_d^2 H_w \times 10^{-3} \text{kVA}. \]

**EXAMPLE: 02**

*Calculate the kVA output of a single phase transformer from the following data:*

\[ \frac{\text{core height}}{\text{distance between core centres}} = 2.8, \quad \frac{\text{diameter of circumscribing circle}}{\text{distance between core centres}} = 0.56, \]

\[ \frac{\text{net iron area}}{\text{area of circumscribing circle}} = 0.7. \]

Current density = 2.3 A/mm², Frequency = 50 Hz, window space factor = 0.27, flux density of core = 1.2 Wb/m², distance between core centres = 0.4 m.

Given Data:

\[
\begin{align*}
H_w/D &= 2.8 \\
K_w &= 0.27 \\
d/D &= 0.56 \\
A/A_c &= 0.7 \\
f &= 50 \text{ Hz} \\
B_m &= 1.2 \text{ wb/m}^2 \\
\delta &= 2.3 \text{ A/mm}^2 \\
D &= 0.4 \text{ m}
\end{align*}
\]
**Solution**

Distance between core centres \( D = 0.4 \) m.
- Core height (window height) \( H_w = 2.8 \times 0.4 = 1.12 \) m.
- Diameter of circumscribing circle \( d = 0.56 \times 0.4 = 0.224 \) m.
- Width of window \( W_w = D - d = 0.4 - 0.224 = 0.176 \) m.
- Area of window \( A_w = H_w \times W_w = 1.12 \times 0.176 = 0.197 \) m\(^2\).
- Area of circumscribing circle \( = \left(\pi/4\right) d^2 = 0.0394 \) m\(^2\).
- Net iron area \( A_i = 0.7 \times 0.0394 = 0.0276 \) m\(^2\).

For a single phase transformer,

\[
Q = 2.22 f B_m K_w \delta A_w A_i \times 10^{-3} \text{kVA} \\
= 2.22 \times 50 \times 1.2 \times 0.27 \times 2.3 \times 10^6 \times 0.197 \times 0.0276 \times 10^{-3} = 450 \text{kVA.}
\]

**EXAMPLE: 03**

Determine the dimensions of core and yoke for a 200 kVA, 50 Hz single phase core type transformer. A cruciform core is used with distance between adjacent limbs equal to 1.6 times the width of core laminations. Assume voltage per turn 14 V, maximum flux density 1.1 Wb/m\(^2\), window space factor 0.32, current density 3 A/mm\(^2\), and stacking factor = 0.9. The net iron area is 0.56 \( d^2 \) in a cruciform core where \( d \) is the diameter of circumscribing circle. Also the width of largest stamping is 0.85 \( d \).

**Given Data:**

- \( Q=200 \text{ kVA} \)
- \( f=50 \text{ Hz} \)
- \( D=1.6 \text{ a} \)
- \( E_t=14 \text{ V} \)
- \( B_m=1.1 \text{ wb/m}^2 \)
- \( K_w=0.32 \)
- \( \delta=3 \text{ A/mm}^2 \)
- \( k_i=0.9 \)
- \( A_i=0.56 \text{ d}^2 \)
- \( a=0.85 \text{ d} \)

**Solution**

Voltage per turn \( E_t = 4.44 f \Phi_m = 4.44 f B_m A_i \).

\[
\therefore \text{Net iron area } A_i = \frac{14}{4.44 \times 50 \times 11} = 0.0573 \text{ m}^2
\]

\[
\therefore \text{Diameter of circumscribing circle } d = \sqrt{\frac{A_i}{0.56}} = \sqrt{\frac{0.0573}{0.56}} = 0.32 \text{ m.}
\]

Width of largest stamping \( a = 0.85 d = 0.85 \times 0.32 = 0.272 \) m.

Distance between core centres \( D = 1.6 a \) (given) = \( 1.6 \times 0.272 = 0.435 \) m.

Width of window \( W_w = D - d = 0.435 - 0.32 = 0.115 \) m.

For a single phase transformer,

\[
Q = 2.22 f B_m K_w \delta A_w A_i \times 10^{-3}
\]

\[
= 2.22 \times 50 \times 1.1 \times 0.32 \times 3 \times 10^6 \times A_w \times 0.573 \times 10^{-8}
\]

\[
\therefore \text{Window area } A_w = 0.0298 \text{ m}^2 \quad \text{Height of window } H_w = 0.0298/0.115 = 0.26 \text{ m.}
\]

Using the same stepped section for the yoke as for core

Depth of yoke \( D_y = a = 0.272 \) m and height of yoke \( H_y = 0.272 \) m.

Overall height of frame \( H = H_w + 2H_y = 26 + 2 \times 0.272 = 0.604 \) m.

Overall length of frame \( W = D + a = 43.5 + 0.272 = 0.737 \) m.
EXAMPLE: 04

Calculate approximate overall dimensions for a 200 kVA, 6600/440 V, 50 Hz, 3 phase core type transformer. The following data may be assumed: emf per turn = 10 V; maximum flux density = 1.3 Wb/m²; current density = 2.5 A/mm²; window space factor = 0.3 overall height = overall width; stacking factor = 0.9. Use a 3 stepped core.

For a three stepped core:
Width of largest stamping = 0.9 d, and
Net iron area = 0.6 d² where d is the diameter of circumscribing circle.

Given Data:

| Q=200 KVA | V_p=6600 V | V_s=440 V | f=50 Hz | E_t=10 V | B_m=1.3 wb/m² |
| δ=2.5 A/mm² | K_w=0.3 | H=W | k_i=0.9 | a=0.9 d | A_i=0.6 d² |

Solution

\[
Net \ iron \ area \ A_t = \frac{E_t}{4.44 \ f \ B_m} = \frac{10}{4.44 \times 50 \times 1.3} = 0.0347 \ m².
\]

Diameter of circumscribing circle \( d = \sqrt{\frac{0.0347}{0.6}} = 0.24 \ m, \)
and width of largest stamping \( a = 0.9 \times 0.24 = 0.216 \ m. \)

Using a 3 stepped section for the yoke
Height of yoke \( H_y = a = 0.216 \ m, \)
Depth of yoke \( D_y = a = 0.216 \ m. \)

for a 3 phase transformer,
\[
Q = \frac{3.33}{B_m K_w} \delta A_w A_t \times 10^{-3}
\]
or
\[
200 = 3.33 \times 50 \times 1.3 \times 0.3 \times 2.5 \times 10^{6} \times A_w \times 0.0347 \times 10^{-8}
\]
\[
\therefore \ Window \ area \ A_w = 0.0355 \ m² \ or \ H_w \times W_w = 0.0355 \ m²
\]

The given condition is, overall height = overall width or \( H = W \)

\[
H = H_w + 2H_y = H_w + 2 \times 0.216 = H_w + 0.432
\]

\[
W = 2D + a = 2(W_w + d) + a = 2W_w + 0.48 + 0.216 = 2W_w + 0.696
\]

As \( H = W, \) we have: \( H_w + 0.432 = 2W_w + 0.696 \)

or \( H_w = 2W_w + 0.264 \)

\[
(2W_w + 0.264) W_w = 0.0355 \ or \ 2W_w^2 + 0.264 W_w - 0.0355 = 0
\]

or width of window \( W_w = 0.083 \ m \) and height of window \( H_w = \frac{0.0355}{0.083} = 0.428 \ m. \)

Thus the dimensions of core are:
Distance between adjacent core centres \( D = W_w + d = 0.823 \ m. \)
Overall height \( H = H_w + 2H_y = 0.86 \ m. \)
Overall width \( W = 2D + a = 0.862 \ m. \)

EXAMPLE: 05

The ratio of flux to full load mmf in a 400 kVA, 50 Hz, single phase core type power transformer is \( 2.4 \times 10^{-6}. \) Calculate the net iron area and the window area of the transformer. Maximum flux density in the core is 1.3 Wb/m², current density 2.7 A/mm² and window space factor 0.26. Also calculate the full load mmf.

Given Data:

- \( Q=400 \) KVA
- \( f=50 \) Hz
- \( φ_m/AT=2.4 \times 10^{-6} \)
- \( B_m=1.3 \) wb/m²
- \( δ=2.7 \) A/mm²
- \( K_w=0.26 \) single phase core type
Solution

\[ K = \frac{\sqrt{4.44 f (\Phi_m / AT) 10^3}}{\sqrt{4.44 \times 50 \times 24 \times 10^{-6} \times 10^3}} = 0.732 \]

Voltage per turn \( E_i = K \sqrt{Q} = 0.732 \sqrt{400} = 14.64 \text{ V} \).

\[ \therefore \text{Flux} \quad \Phi_m = \frac{E_i}{4.44 f} = \frac{14.64}{4.44 \times 50} = 0.066 \text{ Wb.} \]

Net iron area \( A_i = \frac{\Phi_m}{B_m} = \frac{0.066}{1.3} = 0.0507 \text{ m}^2 \).

Window area of single phase transformer

\[ A_w = \frac{Q}{2.22 f B_m K_w \delta A_i \times 10^{-2}} = \frac{400}{2.22 \times 50 \times 1.3 \times 0.26 \times 2.7 \times 10^6 \times 0.0507 \times 10^{-3}} = 0.0777 \text{ m}^2. \]

Full load mmf \( AT = \frac{\Phi_m}{24 \times 10^{-6}} = \frac{0.066}{24 \times 10^{-6}} = 27500 \text{ A.} \)

EXAMPLE: 06

Determine the main dimensions of the core, the number of turns and the cross-section of the conductors for a 5 kVA, 11000/400 V, 50 Hz, single phase core type distribution transformer. The net conductor area in the window is 0.6 times the net cross-section of iron in the core. Assume a square cross-section for the core, a flux density 1 Wb/m², a current density 1.4 A/mm², and a window space factor 0.2. The height of window is 3 times its width.

Given Data:

<table>
<thead>
<tr>
<th>Q=5 KVA</th>
<th>V_p=11000 V</th>
<th>V_s=400 V</th>
<th>f=50 Hz</th>
<th>As=K_wA_w=0.6 A_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_m=1 wb/m²</td>
<td>δ=1.4 A/mm²</td>
<td>K_w=0.2</td>
<td>H_w=3 W_w</td>
<td></td>
</tr>
</tbody>
</table>

Solution

Given that:

Net conductor area = 0.6 \times \text{net iron area} \quad \text{or} \quad K_w A_w = 0.6 A_i

\[ \therefore \text{Window area} \quad A_w = \frac{0.6}{K_w} \delta A_i = \frac{0.6}{0.2} A_i = 3A_i, \]

for a single phase transformer,

\[ Q = 2.22 f B_m K_w \delta A_w A_i \times 10^{-3} \]

\[ 5 = 2.22 \times 50 \times 1.0 \times 0.2 \times 1.4 \times 10^6 \times 3 A_i \times A_i \times 10^{-3} \]

or net iron area \( A_i = 0.00732 \text{ m}^2 \). Gross iron area \( A_{gr} = \frac{0.00732}{0.9} = 0.00814 \text{ m}^2 \).

\[ \therefore \text{Width of core} \quad a = \sqrt{0.00814} \approx 0.09 \text{ m. Gross iron area provided} = 0.0081 \text{ m}^2. \]

Net iron area provided = 0.00729 m², Window area \( A_w = 3 \times 0.00729 = 0.02187 \text{ m}^2 \).

Height of window \( H_w = 3W_w \therefore 3W_w^2 = 0.02187 \). But \( H_w \times W_w = A_w \),

or width of window \( W_w = 0.085 \text{ m}, \) and height of window \( H_w = 3 \times 0.085 = 0.255 \text{ m}. \)

The yoke has the same gross area as the core. Gross area of yoke \( A_y = 0.081 \text{ m}^2 \).
Depth of yoke \( D_y = a = 0.09 \text{ m} \), \( H_y = \frac{0.0081}{0.09} = 0.09 \) m.

Flux \( \Phi_n = B_n A_i = 1.0 \times 72.9 \times 10^{-3} = 7.29 \times 10^{-3} \) Wb.

Voltage per turn \( E_i = 4.44 f \Phi_n = 4.44 \times 50 \times 7.29 \times 10^{-3} = 1.625 \) V.

Primary turns \( \frac{\text{primary voltage}}{\text{secondary voltage}} \times \text{secondary turns} = \frac{11000}{400} \times 246 = 6765 \)

(The turns of the low voltage should be calculated first and that of high voltage afterwards by using the voltage ratio).

Secondary turns \( T_s = \frac{\text{secondary voltage}}{\text{voltage per turn}} = \frac{400}{1.625} = 246 \).

Primary winding current \( I_p = \frac{5000}{11000} = 0.455 \) A.

Area of primary winding conductor \( a_p = \frac{I_p}{\delta} = \frac{0.455}{1.4} = 0.384 \) mm\(^2\).

Using circular conductors, diameter of primary conductor = \( \sqrt{0.324 \times 4} / \pi = 0.642 \) mm

Secondary winding current \( I_s = \frac{5000}{400} = 12.5 \) A.

Area of secondary winding conductor \( a_s = \frac{I_s}{\delta} = \frac{12.5}{1.4} = 8.93 \) mm\(^2\).

Using a square conductor \( 3 \times 3 \) mm\(^2\).

Overall dimensions of core:
- Distance between core centres \( D = a + W_w = 0.09 + 0.085 = 0.175 \) m.
- Length of frame \( W = D + a = 0.175 + 0.09 = 0.265 \) m.
- Height of frame \( H = H_w + 2H_y = 0.255 + 2 \times 0.09 = 0.435 \) m.

EXAMPLE: 07

Calculate the main dimensions and winding details of a 100 kV A 2000/400 volt, 50 Hz, single phase shell type, oil immersed, self cooled transformer. Assume:
- Voltage per turn, 10 V flux density in core, 1.1 Wb/m\(^2\); current density 2 A/mm\(^2\) window space factor, 0.33.
- The ratio of window height to window width 3 and ratio of core depth to width of central limb = 2.5. The stacking factor is 0.9.

Given data
- \( Q = 100 \text{ KVA} \)
- \( V_p = 2000 \text{ V} \)
- \( V_s = 400 \text{ V} \)
- \( f = 50 \text{ Hz} \)
- \( E_t = 10 \text{ V} \)
- \( K_i = 0.9 \)
- \( B_m = 1.1 \text{ wb/m}^2 \)
- \( \delta = 2 \text{ A/mm}^2 \)
- \( K_w = 0.33 \)
- \( H_w/W_w = 3 \)
- \( b/2a = 2.5 \)

Solution

Net iron area \( A_i = \frac{E_t}{4.44 f B_m} = \frac{10}{4.44 \times 50 \times 1.1} = 0.441 \text{ m}^2 \).

Gross iron area \( A_{gi} = \frac{0.041}{0.9} = 0.0555 \text{ m}^2 \).

\( \frac{b}{2a} = 2.5 \) (given), We have, gross iron area \( A_{gi} = 2a \times b \).
Core depth \( b = 2.5 \times 0.135 = 0.3475 \) m.
The yoke carries half of the flux in the central limb. Assuming the same flux density in the core as in the limb, the area of yoke is equal to half the area of the central limb.

Gross area of yoke \( A_y = \frac{0.04555}{2} = 227.75 \times 10^{-3} \) m².

Depth of yoke \( D_y = b = 0.3375 \) m. \[ \therefore \text{Height of yoke } H_y = \frac{22.75 \times 10^{-3}}{0.3375} = 0.0675 \text{ m.} \]

The side limbs carry half of the flux in the central limb. Therefore, the width of side limb is half of the width of central limb. Width of side limb \( a = 0.0675 \) m.

for a single phase transformer,
\[
Q = 2.22 f B_m K_w \delta A_w A_w \times 10^{-3} \\
100 = 2.22 \times 50 \times 1.1 \times 0.33 \times 2 \times 10^6 \times 0.041 \times A_w \times 10^{-3}
\]

We have, \( H_w \times W_w = 0.0303 \) and \( \frac{H_w}{W_w} = 3. \)

\( \therefore \) Window area \( A_w = 0.0303 \) m².

or \( 3W_w^2 = 303 \times 10^{-4} \). Thus width of window \( W_w = 0.1 \) m.

Height of window \( H_w = 0.3 \) m

Overall height of frame \( H = H_w + 2H_y = 0.3 + 2 \times 0.0675 = 0.435 \) m.

Overall length of frame \( W = 2W_w + 4a = 2 \times 0.1 + 4 \times 0.0675 = 0.47 \) m.

Overall depth of frame \( b = 0.3375 \) m.

**Windings**

H.V. winding turns \( T_p = \frac{2000}{10} = 200, \)

L.V. winding turns \( T_s = \frac{400}{10} = 400. \)

H.V. winding current \( I_p = \frac{100 \times 1000}{2000} = 50 \) A.

H.V. winding conductor area \( a_p = \frac{50}{2} = 25 \) mm².

L.V. winding current \( I_s = \frac{100 \times 1000}{400} = 250 \) A.

L.V. winding area \( a_s = \frac{250}{2} = 125 \) mm².

**EXAMPLE: 08**

Calculate the core and window areas required for a 1000 kVA, 6600/400 V, 50Hz, single phase core type transformer. Assume a maximum flux density of 1.25 Wb/m² and a current density of 2.5 A/mm². Voltage per turn = 30 V. Window space factor = 0.32.

**Given data**

\[
\begin{align*}
\text{kVA} &= 1000 & \text{f} &= 50 \text{ Hz} & \text{B}_m &= 1.25 \text{ Wb/m}^2 & \text{1-phase} \\
\text{V}_p &= 6600\text{V} & \text{V}_s &= 400\text{V} & \delta &= 2.5 \text{ A/mm}^2 \\
\text{E}_t &= 30\text{V} & \text{K}_w &= 0.32 & \text{Core type}
\end{align*}
\]
Solution

Emf per turn, $E_t = 4.44 \, f \phi_m$

$\therefore \phi_m = \frac{E_t}{4.44 \, f} = \frac{30}{4.44 \times 50} = 0.1351 \, \text{Wb}$

Flux density, $B_m = \frac{\phi_m}{A_i}$

$\therefore$ The net area of cross-section of core

$A_i = \frac{\phi_m}{B_m} = \frac{0.1351}{1.25} = 0.108 \, \text{m}^2 = 0.108 \times 10^6 \, \text{mm}^2$

The kVA rating of transformer, $Q = 2.22 \, f \, B_m \, A_i \, K_w \, A_w \delta \times 10^{-3}$

$\therefore$ Window area, $A_w = \frac{Q}{2.22 \, f \, B_m \, A_i \, K_w \, \delta \times 10^{-3}}$

$= \frac{1000}{2.22 \times 50 \times 1.25 \times 0.108 \times 0.32 \times 2.5 \times 10^6 \times 10^{-3}}$

$= 0.0834 \, \text{m}^2$

$= 0.0834 \times 10^6 \, \text{mm}^2$

Result

Net core area, $A_i = 0.108 \, \text{m}^2 = 0.108 \times 10^6 \, \text{mm}^2$

Window area, $A_w = 0.0834 \, \text{m}^2 = 0.0834 \times 10^6 \, \text{mm}^2$

EXAMPLE: 09

Estimate the main dimensions including winding conductor area of a 3-phase, Δ-Y core type transformer rated at 300 kVA, 6600/440 V, 50Hz. A suitable core with 3-steps having a circumscribing circle of 0.25 m diameter and a leg spacing of 0.4 m is available. Emf per turn = 8.5V, $\delta = 2.5 \, \text{A/mm}^2$, $K_w = 0.28$, $S_r = 0.9$ (stacking factor).

Given data

<table>
<thead>
<tr>
<th>3-phase, Δ - Y</th>
<th>50 Hz</th>
<th>$E_t = 8.5 , \text{V}$</th>
<th>leg spacing = 0.4m</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-stepped core</td>
<td>$\delta = 2.5 , \text{A/mm}^2$</td>
<td>Core type</td>
<td>$S_r = 0.9$</td>
</tr>
<tr>
<td>300 kVA</td>
<td>$d = 0.25$</td>
<td>$K_w = 0.28$</td>
<td>6600/440V</td>
</tr>
</tbody>
</table>

Solution

Let 440V side be secondary and 6600V be primary. Here the secondary is star connected and primary is delta connected.

$\therefore$ Secondary voltage per phase, $V_s = \frac{440}{\sqrt{3}} = 254 \, \text{V}$
Also, \( E_s \approx V_s \)

Emf per turn, \( E_t = E_s / T_s \)

\[ T_s = \frac{E_s}{E_t} = \frac{259}{8.5} = 29.88 \approx 30 \text{ turns} \]

\[ \therefore \text{Number of primary turns per phase} \]

\[ T_p = T_s \times \frac{V_p}{V_s} = 30 \times \frac{6600}{254} = 779.5 \approx 780 \text{ turns} \]

The phase voltage ratio of transformer, \( \frac{V_s}{V_p} = \frac{254}{6600} \)

\[ \therefore \text{Number of primary turns per phase} \]

The kVA rating of transformer, \( Q = \sqrt{3} V_{LP} I_{LP} \times 10^{-3} = \sqrt{3} V_{LS} I_{LS} \times 10^{-3} \)

where,

- \( V_{LP} \) = Line voltage on primary side
- \( I_{LP} \) = Line current on primary side
- \( V_{LS} \) = Line voltage on secondary side
- \( I_{LS} \) = Line current on secondary side

Line current on primary side,

\[ I_{LP} = \frac{Q}{\sqrt{3} V_{LP} \times 10^{-3}} \]

\[ = \frac{300}{\sqrt{3} \times 6600 \times 10^{-3}} = 26.24 \text{ A} \]

Since primary is delta connected,

The phase current on primary,

\[ I_p = \frac{I_{LP}}{\sqrt{3}} = \frac{26.24}{\sqrt{3}} = 15.15 \text{ A} \]

The area of cross-section of primary conductor,

\[ a_p = \frac{I_p}{\delta} = \frac{15.15}{2.5} = 6.06 \text{ mm}^2 \]

The line current on secondary sides,

\[ I_{LS} = \frac{Q}{\sqrt{3} \times V_{LS} \times 10^{-3}} \]

\[ = \frac{300}{\sqrt{3} \times 440 \times 10^{-3}} = 393.65 \text{ A} \]

Since secondary is star connected,

The phase current on secondary,

\[ I_s = I_{LS} = 393.65 \text{ A} \]

The area of cross-section of secondary conductor,

\[ a_s = \frac{I_s}{\delta} = \frac{393.65}{2.5} = 157.5 \text{ mm}^2 \]
The copper area in window, \[ A_c = 2 \left( a_p T_p + a_s T_s \right) \]
\[ = 2 \left( 6.06 \times 780 + 157.5 \times 30 \right) \]
\[ = 1890.36 \text{ mm}^2 \]

Window Area, \[ A_w = \frac{A_c}{K_w} = \frac{1890.36}{0.28} = 67512.86 \text{ mm}^2 \]
\[ = 67512.86 \times 10^{-6} \text{ m}^2 = 0.0675 \text{ m}^2 \]

Area of circumscribing circle \[ = \frac{\pi d^2}{4} = \frac{\pi (0.25)^2}{4} = 0.049 \text{ m}^2 \]

For 3 stepped core, the ratio \[ \frac{\text{Gross core area}}{\text{Area of circumscribing circle}} = 0.84 \]

Gross core area, \[ A_{gi} = 0.84 \times \text{area of circumscribing circle} \]
\[ = 0.84 \times 0.049 = 0.041 \text{ m}^2 \]

Net core area, \[ A_i = S_r \times A_{gi} \]
\[ = 0.9 \times 0.041 = 0.0369 \text{ m}^2 = 0.037 \times 10^6 \text{ mm}^2 \]

Given that, leg spacing = 0.45 m

Width of window, \[ W_w = \text{leg spacing} = 0.45 \text{ m} \]

Height of window, \[ H_w = \frac{A_w}{W_w} = \frac{0.0675}{0.45} = 0.15 \text{ m} \]

Result

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of primary turns per phase, ( T_p )</td>
<td>780</td>
</tr>
<tr>
<td>Number of secondary turns per phase, ( T_s )</td>
<td>30</td>
</tr>
<tr>
<td>Area of cross-section of primary conductor, ( a_p )</td>
<td>6.06 mm(^2)</td>
</tr>
<tr>
<td>Area of cross-section of secondary conductor, ( a_s )</td>
<td>157.5 mm(^2)</td>
</tr>
<tr>
<td>Net core area, ( A_i )</td>
<td>0.0369 m(^2)</td>
</tr>
<tr>
<td>Window area, ( A_w )</td>
<td>0.0675 m(^2)</td>
</tr>
<tr>
<td>Height of window, ( H_w )</td>
<td>0.15 m</td>
</tr>
<tr>
<td>Width of window, ( W_w )</td>
<td>0.45 m</td>
</tr>
</tbody>
</table>

EXAMPLE: 10

Determine the dimensions of core and window for a 5 kVA, 50 Hz, 1-phase, core type transformer. A rectangular core is used with long side twice as long as short side. The window height is 3 times the width. Voltage per turn = 1.8 V. Space factor = 0.2. \( \delta = 1.8 \text{ A/mm}^2 \). \( B_w = 1 \text{ Wb/m}^2 \).
**Given Data**

- \( Q = 5 \text{kVA} \)
- \( f = 50 \text{ Hz} \)
- 1-phase
- \( E_i = 1.8 \text{ V} \)
- \( A \) = 3
- \( A_w = 3W_w \)
- \( \delta = 1.8 \text{ A/mm}^2 \)
- \( B_m = 1 \text{ Wb/m}^2 \)

**Solution**

Emf per turn, \( E_t = 4.44 f \phi_m \)

\[ \phi_m = \frac{E_t}{4.44 f} \]

\[ = \frac{1.8}{4.44 \times 50} = 0.0081 \text{ Wb} \]

Net core area, \( A_i = \frac{\phi_m}{B_m} = \frac{0.0081}{1} = 0.0081 \text{ m}^2 \)

Gross core Area, \( A_g = \frac{A_i}{S_f} = \frac{0.0081}{0.9} = 0.009 \text{ m}^2 \)

Cross-section of the core is rectangle. Hence, \( A_g = \text{length} \times \text{breadth} = a \times b \)

Given that \( a = 2b \)

\[ b = \sqrt{\frac{A_g}{2}} = \sqrt{\frac{0.009}{2}} = 0.067 \text{ m} \]

\[ a = 2b = 2 \times 0.067 = 0.134 \text{ m} \]

kVA rating of 1-phase transformer

\[ Q = 2.22 f B_m A_i K_w A_w \delta \times 10^{-3} \]

Window area, \( A_w = \frac{Q}{2.22 f B_m A_i K_w \delta \times 10^{-3}} \)

\[ = \frac{5}{2.22 \times 50 \times 1 \times 0.0081 \times 0.2 \times 1.8 \times 10^6 \times 10^{-3}} \]

\[ = 0.0154 \text{ m}^2 \]

Also, Window area, \( A_w = H_w W_w \)

Given that, \( H_w = 3W_w \)

Hence, \( A_w = H_w W_w = 3W_w \times W_w = 3W_w^2 \)

\[ W_w = \sqrt{\frac{A_w}{3}} = \sqrt{\frac{0.0154}{3}} = 0.0716 \text{ m} \]

\[ H_w = 3 W_w = 3 \times 0.0716 = 0.2148 \text{ m} \]

**Result**

- The Net core area, \( A_i = 0.0081 \text{ m}^2 \)
- The dimensions of the core, \( a \times b = 0.134 \times 0.067 \text{ m} \)
- The window area, \( A_w = 0.0154 \text{ m}^2 \)
- The dimensions of window, \( H_w \times W_w = 0.2148 \times 0.0716 \text{ m} \)
EXAMPLE: 11

Determine the dimensions of the core, the number of turns, the cross-section area of conductors in primary and secondary windings of a 100 kVA, 2200/480 V, 1-phase, core type transformer, to operate at a frequency of 50 Hz, by assuming the following data. Approximate Volt per turn = 7.5 Volt. Maximum flux density = 1.2 Wb/m². Ratio of effective cross-sectional area of core to square of diameter of circumscribing circle is 0.6. Ratio of height to width of window is 2. Window space factor = 0.28. Current density = 2.5 A/mm²

**Given data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 kVA</td>
<td></td>
</tr>
<tr>
<td>2200/480 V</td>
<td></td>
</tr>
<tr>
<td>K_w = 0.28</td>
<td></td>
</tr>
<tr>
<td>Core type</td>
<td></td>
</tr>
<tr>
<td>E = 7.5 V</td>
<td></td>
</tr>
<tr>
<td>A_i/d² = 0.6</td>
<td></td>
</tr>
<tr>
<td>B_m = 1.2 Wb/m²</td>
<td></td>
</tr>
<tr>
<td>H_w/W_w = 2</td>
<td></td>
</tr>
</tbody>
</table>

**Solution**

Emf per turn, \( E_t = 4.44 f \phi_m \)

\[ \phi_m = \frac{E_t}{4.44 f} = \frac{7.5}{4.44 \times 50} = 0.03378 \text{ Wb} \]

Also, \( B_m = \frac{\phi_m}{A_i} \)

\[ A_i = \frac{\phi_m}{B_m} = \frac{0.03378}{1.2} = 0.0282 \text{ m}^2 \]

Given that, \( \frac{A_i}{d^2} = 0.6 \). Hence the core is 3-stepped core

\[ d = \sqrt{\frac{A_i}{0.6}} = \sqrt{\frac{0.0282}{0.6}} = 0.2168 \text{ m} \]

The kVA rating of 1-phase transformer

\[ Q = 2.22 f B_m A_i K_w A_w \delta \times 10^{-3} \]

\[ A_w = \frac{Q}{2.22 f B_m A_i K_w \delta \times 10^{-3}} = \frac{100}{2.22 \times 50 \times 1.2 \times 0.0282 \times 0.28 \times 2.5 \times 10^{-6} \times 10^{-3}} = 0.038 \text{ m}^2 \]

Given that, \( \frac{H_w}{W_w} = 2 \), \( H_w = 2 W_w \)

Window area, \( A_w = H_w W_w = 2 W_w \times W_w \)

Width of window, \( W_w = \sqrt{\frac{A_w}{2}} = \sqrt{\frac{0.0382}{2}} = 0.1378 \text{ m} \)

Height of window, \( H_w = 2 W_w = 2 \times 0.1378 = 0.2756 \text{ m} \)

Let 480V side be secondary and 2200V side be primary.
\[ \therefore \text{Secondary voltage, } V_s = 480 \text{V} \]

Also, \( E_s \approx V_s \)

\[ \text{Emf per turn, } E_t = \frac{E_s}{T_s} \]

\[ \therefore \text{Number of turns in secondary, } T_s = \frac{E_s}{E_t} = \frac{V_s}{7.5} = 64 \text{ turns} \]

\[ \text{The voltage ratio of transformers, } \frac{V_s}{V_p} = \frac{480}{2200} \]

\[ \therefore \text{Number of turns in primary, } T_p = T_s \times \frac{V_p}{V_s} = 64 \times \frac{2200}{480} = 293 \text{ turns} \]

\[ \text{The } kVA \text{ rating of single phase transformer } \]
\[ Q = V_p I_p \times 10^{-3} = V_s I_s \times 10^{-3} \]

\[ \therefore \text{Current in primary, } I_p = \frac{Q}{V_p \times 10^{-3}} = \frac{100}{2200 \times 10^{-3}} = 45.45 \text{ A} \]

\[ \text{Area of cross-section of primary conductor } \]
\[ a_p = \frac{I_p}{\delta} = \frac{45.45}{2.5} = 18.1818 \text{ mm}^2 \]

\[ \text{Current in secondary, } I_s = \frac{Q}{V_s \times 10^{-3}} = \frac{100}{480 \times 10^{-3}} = 208.33 \text{ A} \]

\[ \text{Area of cross-section of secondary conductor } \]
\[ a_s = \frac{I_s}{\delta} = \frac{208.33}{2.5} = 83.33 \text{ mm}^2 \]

**Result**

- Net core Area, \( A_l = 0.0282 \text{ m}^2 \)
- Diameter of circumscribing circle, \( d = 0.2168 \text{ m} \)
- Window Area, \( A_w = 0.038 \text{ m}^2 \)
- Window dimension, \( H_w \times W_w = 0.2756 \times 0.1378 \text{ m} \)
- Number of turns in primary, \( T_p = 293 \text{ turns} \)
- Number of turns in secondary, \( T_s = 64 \text{ turns} \)
- Area of cross-section of primary conductor, \( a_p = 18.18 \text{ mm}^2 \)
- Area of cross-section of secondary conductor, \( a_s = 83.33 \text{ mm}^2 \)
OPERATING CHARACTERISTICS

REGULATION

Fig. (a) shows an approximate equivalent circuit of transformer with parameters referred to primary side.

\[ V_p' = V_p + I_p R_p \cos \phi + I_p X_p \sin \phi \]

\[ \varepsilon = \varepsilon_r \cos \phi + \varepsilon_p \sin \phi \]

(a) Simplified equivalent circuit and phasor diagram with lagging load.

On no load the secondary terminal voltage \( V_p' = V_p \). The drop in secondary terminal voltage from no load to full load can be calculated by drawing the phasor diagram. Fig. (b) shows the phasor diagram at a lagging power factor \( \cos \phi \).

EXAMPLE: 01

Estimate the per unit regulation, at full load and 0.8 power factor aging, for a 300 kVA, 50 Hz, 6600/400 V, 3 phase, delta/star, core type transformer. The data given is:

H.V. winding—

Outside diameter = 0.36 m, inside diameter = 0.29 m, area of conductor = 5.4 mm².

L.V. winding—

Outside diameter = 0.26 m, inside diameter = 0.22 m, area of conductor = 170 mm².

Length of coils = 0.5 m, voltage per turn = 8 V, resistivity = 0.21 Ω/m/mm².

Solution

L.V. voltage per phase \( V_s = \frac{400}{\sqrt{3}} = 231 \) V.

L.V. turns per phase \( T_s = \frac{V_s}{E_t} = \frac{231}{8} = 29 \). H.V. voltage per phase \( V_p = 6600 \) V.

H.V. turns per phase \( T_p = \frac{6600}{231} \times 29 = 826 \)

Mean diameter of l.v winding = \( \frac{\text{outside diameter} + \text{inside diameter}}{2} \)

Mean diameter of l.v. winding \( = \frac{0.26 + 0.22}{2} = 0.24 \) m.
Length of mean turn of l.v. winding = \( \pi \times \text{mean diameter} \)

\[
\text{Length of mean turn of l.v. winding} = \pi \times 0.24 = 0.752 \text{ m.}
\]

Resistance of l.v. winding = \( \frac{\rho L_{\text{mean}} T_{s}}{a} \)

Resistance of l.v. winding = \( r_{s} = \frac{0.021 \times 29 \times 0.752}{170} = 0.00269 \text{ \Omega} \)

Mean diameter of h.v. winding = \( \frac{\text{outside diameter} + \text{inside diameter}}{2} \)

Mean diameter of h.v. winding = \( (0.36 + 0.29)/2 = 0.325 \text{ m.} \)

Length of mean turn of h.v. winding = \( \pi \times \text{mean diameter} \)

\[
\text{Length of mean turn of h.v. winding} = \pi \times 0.325 = 1.02 \text{ m.}
\]

Resistance of h.v. winding = \( \frac{\rho L_{T} T_{\text{s}}}{a_{p}} \)

\[
\text{Resistance of h.v. winding} = \frac{0.021 \times 826 \times 102}{54} = 3.28 \text{ \Omega}
\]

Resistance of transformer referred to primary = \( R_{p} = r_{s} + r_{p} \left( \frac{T_{p}}{T} \right)^{2} \)

Resistance of transformer referred to primary = \( R_{p} = 3.28 + 0.00269 \times (826/29)^{2} = 5.47 \text{ \Omega} \)

H.V. winding current per phase \( I_{p} = \frac{kVA \times 1000}{3V_{p}} \)

\[
\text{H.V. winding current per phase} I_{p} = \frac{300 \times 1000}{3 \times 6600} = 15.1 \text{ A.}
\]

\[
\text{p.u. resistance} = \frac{I_{R}}{V_{p}} \quad \Rightarrow \quad \text{P.U. resistance} = \frac{15.1 \times 5.47}{6600} = 0.0126.
\]

Mean diameter = \( \frac{\text{outside diameter of h.v} + \text{inside diameter of l.v.}}{2} \)

Mean diameter = \( (0.36 + 0.22)/2 = 0.29 \text{ m.} \)

Length of mean turn = \( \pi \times \text{mean diameter} \)

\[
\text{Length of mean turn} L_{mt} = \pi \times 0.29 = 0.91 \text{ m}
\]

Width of l.v. winding \( b_{s} = (0.26 - 0.22)/2 = 0.02 \text{ m.} \)

Width of h.v. winding \( b_{p} = (0.36 - 0.29)/2 = 0.035 \text{ m.} \)

Width of duct \( a = (0.29 - 0.26)/2 = 0.015 \text{ m.} \)

Leakage reactance of transformer referred to primary side

\[
X_{p} = \pi f \mu_{0} T_{r}^{2} \frac{L_{mt}}{L_{c}} \left( a + \frac{b_{p} + b_{s}}{3} \right)
\]

\[
X_{p} = 2\pi \times 50 \times 4\pi \times 10^{-7} \times (826)^{2} \times \frac{0.91}{0.5} \left( \frac{0.015 + 0.035 + 0.02}{3} \right) = 17.3 \text{ \Omega}
\]
p.u. leakage reactance $\frac{L}{V_p} = \frac{151 \times 1.73}{6600} = 0.0395$

per unit regulation

\[ \varepsilon = \varepsilon_p \cos \phi + \varepsilon_x \sin \phi = 0.0126 \times 0.8 + 0.0395 \times 0.6 = 0.0388. \]

**NO LOAD CURRENT**

1. **No load current of single phase transformer**

   The no load current $I_0$ consists of two components: (i) magnetizing current $I_m$ (ii) loss component $I_l$ and its value is given by $I_0 = (I_m^2 + I_l^2)^{1/2}$.

   Thus the estimation of no load current $I_0$ requires the calculation of its two components $I_m$ and $I_l$.

   The calculation of total mmf is based upon the maximum value of flux density.

   \[ \therefore \text{Rms value of magnetising current } I_m = AT_0 / \sqrt{2} T_p \]

   But the magnetising is not sinusoidal and therefore the peak factor $K_{pk}$ should be used in place of $\sqrt{2}$.

   \[ K_{pk} = \text{Amplitude factor} \times \sqrt{3} \]

   \[ I_m = AT_d (K_{pk} T_p) \]

   Let the iron losses be $P_i$, the loss component

   \[ I_l = P_i / V_n \]

   No load current $I_0 = I_m \frac{L}{2l} \frac{I_{l}}{2l}$

2. **No load current of three phase transformer**

   \[ \therefore \text{Magnetising current per phase } I_m = AT_d (\sqrt{2} T_p) \text{ or } I_m = AT_d (K_{pk} T_p) \]

   Let $P_i$ be the total iron loss for the three phases.

   \[ \therefore I_l = P_i / 3V_p \]

   No load current $I_0 = \sqrt{I_m^2 + I_l^2}$
EXAMPLE: 01

A single phase, 400 V, 50 Hz, transformer is built from stampings having a relative permeability of 1000. The length of the flux path is 2.5 m, the area of cross-section of the core is $2.5 \times 10^{-3}$ m$^2$ and the primary winding has 800 turns. Estimate the maximum flux and no load current of the transformer. The iron loss at the working flux density is 2.6 W/kg. Iron weighs 5.8 $\times$ $10^5$ kg/m$^3$. Stacking factor is 0.9.

Given Data:

\[
\begin{align*}
E_p &= 400 \text{ V} & f &= 50 \text{ Hz} & \mu_r &= 1000 & l_i &= 2.5 \text{ m} & A_p &= 2.5 \times 10^{-3} & T_p &= 800 \\
\rho &= 2.6 \text{ W/kg} & \text{Density of iron} &= 7.8 \times 10^3 \text{ kg/m}^3 & K_i &= 0.9
\end{align*}
\]

**Solution.** Net iron area $A_i = 0.9 \times 2.5 \times 10^{-3} = 2.25 \times 10^{-3}$ m$^2$

We have, $E_p = 4.44 f \Phi_m T_p = 4.44 f B_m A_i T_p$.

\[
\therefore \quad B_m = \frac{400}{4.44 \times 50 \times 2.25 \times 10^{-3} \times 800} = 1.0 \text{ Wb/m}^2.
\]

\[
\therefore \text{ Flux in the core } = B_m A_i = 1 \times 2.25 \times 10^{-3} = 2.25 \times 10^{-3} \text{ Wb}.
\]

Magnetizing mmf

\[
AT_0 = \text{ reluctance} \times \text{ flux} = \frac{l_i}{\mu A_i} \times \Phi_m = \frac{l_i}{\mu_r \mu_0} B_m
\]

\[
= \frac{25 \times 1}{1000 \times 4\pi \times 10^{-7}} = 1980 \text{ A}.
\]

Magnetising current

\[
I_m = \frac{AT_0}{\sqrt{2} T_p} = \frac{1980}{\sqrt{2} \times 800} = 1.75 \text{ A}.
\]

Volume of core

\[
= 2.25 \times 10^{-3} \times 2.5 = 5.625 \times 10^{-3}.
\]

\[
= \text{ Weight of core} = 5.625 \times 10^{-3} \times 7.8 \times 10^3 = 43.8 \text{ kg and iron loss} = 2.6 \times 43.8 = 144 \text{ W}.
\]

Loss component of no load current $I_l = \frac{P_l}{V_p} = \frac{114}{400} = 0.25 \text{ A}$.

\[
\therefore \text{ No load current} \quad I_0 = \sqrt{1.75^2 + 0.285^2} = 1.77 \text{ A}.
\]

EXAMPLE: 02

A 6600 V, 60 Hz single phase transformer has a core of sheet steel. The net iron cross-sectional area is $22.6 \times 10^{-3}$ m$^2$, the mean length is 2.23 m, and there are four lap joints. Each lap joint takes 1/4 times as much reactive mmf as is required per metre of core. If $B_m = 1.1$ Wb/m$^2$, determine (a) the number of turns on the 6600 V winding and (b) the no load current. Assume an amplitude factor of 1.52 and that for given flux density, mmf per metre = 232 A/m; specific loss = 1.76 W/kg. Specific gravity of plates = 7.5.

Given Data:

\[
\begin{align*}
E_p &= 6600 \text{ V} & f &= 60 \text{ Hz} & A_i &= 22.6 \times 10^{-3} \text{ m}^2 & \text{lap joint} &= 4 \text{ Specific gravity} &= 7.5 \times 10^{-3} \text{ Kg/m}^3 & l_i &= 2.23 \text{ m} \\
\text{mmf for joint} &= 1/4 \times \text{mmf/m of core} & \text{mmf/m of core} &= 232 & \text{specific loss} &= 1.76 \text{ W/kg}
\end{align*}
\]
Solution. (a) Number of turns \( T = \frac{E}{4.44 f B_m A_i} = \frac{6600}{4.44 \times 60 \times 11 \times 22.6 \times 10^{-3}} = 1100 \). 

(b) Mmf required for iron parts \( = 232 \times 2.23 = 517 \text{ A} \). 

Mmf required for joints \( = 4 \times \frac{1}{4} \times 232 = 232 \text{ A} \). 

Total magnetizing mmf \( AT_0 = 517 + 232 = 749 \text{ A} \). 

Magnetizing current \( I_m = \frac{AT_0}{K_{pk} T_p} = \frac{749}{1.52 \times \sqrt{2} \times 1100} = 0.318 \text{ A} \). 

(as peak factor \( K_{pk} = \text{amplitude factor} \times \sqrt{2} \). 

Weight of core \( = 2.23 \times 22.6 \times 10^{-3} \times 7.5 \times 10^3 = 378 \text{ kg} \). 

Total iron loss \( P_i = 1.76 \times 378 = 665 \text{ W} \). Loss component \( I_i = \frac{665}{6600} = 0.1 \text{ A} \). 

No load current \( I_0 = \sqrt{(0.318)^2 + (0.1)^2} = 0.333 \text{ A} \). 

EXAMPLE: 03

Calculate the active and reactive components of no load current of a 4100 V, 50 Hz, single phase transformer having the following particulars:

Core of transformer steel; Stacking factor = 0.9; density = \( 7.8 \times 10^3 \text{ kg/m}^3 \); length of mean flux path 2.2 m; gross iron section \( 10 \times 10^{-3} \text{ m}^2 \); primary turns 200; joints equivalent to 0.2 mm air gap. Use the following data:

<table>
<thead>
<tr>
<th>( B_m )</th>
<th>( \text{Wb/m}^2 )</th>
<th>0.9</th>
<th>1.0</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mmf ( A/m )</td>
<td>130</td>
<td>2.0</td>
<td>420</td>
<td>660</td>
<td>1300</td>
<td></td>
</tr>
<tr>
<td>Iron loss ( W/kg )</td>
<td>0.8</td>
<td>1.3</td>
<td>1.9</td>
<td>2.4</td>
<td>2.9</td>
<td></td>
</tr>
</tbody>
</table>

Solution. Gross iron area \( = 10 \times 10^{-3} \text{ m}^2 \). Net iron area \( = 9 \times 10^{-3} \text{ m}^2 \).

Flux in core \( \Phi_m = \frac{400}{4.44 \times 50 \times 200} = 9.02 \times 10^{-3} \text{ Wb} \).

Flux density \( B_m = \frac{\Phi_m}{A_i} = 9.02 \times 10^{-3} \times 9 \times 10^{-3} = 1 \text{ Wb/m}^2 \).

Corresponding to \( B_m = 1.0 \text{ Wb/m}^2 \), mmf/metre = 210 A, loss per kg = 1.3 W.

\( \therefore \) Mmf for iron path \( = 210 \times 2.2 = 462 \text{ A} \).

Mmf for joints \( = 800,000 B'_g = 800,000 \times 1 \times \frac{0.2}{1000} = 160 \text{ A} \).

\( \therefore \) Total magnetising mmf, \( AT_0 = 462 + 160 = 622 \text{ A} \).

\( \therefore \) Reactive (magnetising) current, \( I_m = \frac{622}{\sqrt{2} \times 260} = 2.2 \text{ A} \).

Volume of core \( = 2.2 \times 0.009 = 0.0198 \text{ m}^3 \).

Weight of core \( = 7.8 \times 10^3 \times 0.0198 = 155 \text{ kg} \). Total iron loss \( = 155 \times 1.3 = 201.5 \text{ W} \).

Loss (active) component of no load current \( I_i = \frac{201.5}{400} = 0.5 \text{ A} \).

\( \therefore \) No load current \( = \sqrt{I_m^2 + I_i^2} = \sqrt{2.2^2 + 0.5^2} = 2.26 \text{ A} \).
TEMPERATURE RISE IN TRANSFORMERS

The problem of temperature rise and cooling of transformers is essentially the same as that of rotating machinery. Similar to the latter, the losses developed in the transformer cores and windings during conversion are converted into thermal energy and cause heating of corresponding transformer parts. From its source the heat is directed, due to thermal gradients, to the places where it may be transferred to a cooling medium i.e. to air, or water, depending upon the method of transformer cooling. Heat dissipation occurs in the same way as in electrical machines, i.e. by way of radiation and convection.

The path of heat flow is:

(i) From the internal most heated spots of a given part (of core or winding) to their outer surfaces in contact with the oil.
(ii) From the outer surface of a given transformer part to the oil that cools it.
(iii) From the oil to the walls of a cooler, for instance, of the tank.
(iv) From the walls of the cooler to the cooling medium—air or water.

In section (i) the heat is transferred by conduction. In sections (ii) and (iii), the heat is transferred by convection of the oil.
In section (iv), the heat is dissipated by both convection and radiation.

Transformer Oil as a Cooling Medium

Tests have shown that an average working temperature of the oil \( \theta_o \) = 50 to 60°C and oil viscosity corresponding to this temperature, the specific heat dissipation due to convection of oil is

\[ \lambda_{\text{conv}} = 40.3(\theta/10)^{1/4} \text{ W/m}^2 - ^\circ\text{C} \]

where \( \theta \) = temperature difference of the surface relative to oil, \( ^\circ\text{C} \)
\( H \) = height of dissipating surface, m.

If we assume an average \( \theta = 20^\circ\text{C} \) and \( H = 0.5 \) to 1 m, \( \lambda_{\text{conv}} = 80 \) to 100 W/m\(^2\) - \( ^\circ\text{C} \).

The corresponding figure for convection due to air is 8 W/m\(^2\) - \( ^\circ\text{C} \). Thus the convection due to oil is 10 times and above that with air. This constitutes a major valuable property of oil as a cooling medium.

Temperature Rise in Plain Walled Tanks

The walls of tank dissipate heat by both radiation and convection. It has been found experimentally that a plain tank surface dissipates 6 and 6.5 W/m\(^2\) - \( ^\circ\text{C} \) by radiation and convection respectively (for a temperature rise of nearly 40°C above an ambient temperature of 20°C. Thus the total loss dissipation is 12.5 W/m\(^2\) - \( ^\circ\text{C} \).

\[ \therefore \text{Temperature rise } \theta = \frac{\text{total loss}}{\text{specific heat dissipation} \times \text{surface}} = \frac{P_t + P_c}{125 S_t} \]

where \( S_t \) = heat dissipating surface of tank. The surface, to be considered in applying the above formula, is the total area of the vertical sides plus one half area of the cover, unless the oil is in contact with the cover in which case whole area of the lid should be taken. The area of bottom of the tank should be neglected as it has very little cooling effect.

For transformers of low output, the plain walled tanks large are enough to accommodate the transformer and oil have sufficient surface to keep the temperature rise within limits. But for transformers of large output, the plain walled tanks are not sufficient to dissipate losses.
This is because volume and hence losses increase as cube of linear dimensions while the dissipating surface increases as the square of linear dimensions. Thus an increase in rating results in an increase in loss to be dissipated per unit area giving a higher temperature rise.

Modern oil immersed power transformers with natural oil cooling and a plain tank may be produced for outputs not exceeding 20—30 kVA. Transformers rated for larger outputs must be provided with means to improve the conditions of heat dissipation. This may be done by providing corrugations, tubing or radiators where feasible.

**DESIGN OF TANK AND COOLING TUBES OF TRANSFORMER**

The transformers are provided with cooling tubes to increase the heat dissipating area. The tubes are mounted on the vertical sides of the transformer tank. But the increase in dissipation of heat is not proportional to increase in area, because the tubes would screen (conceals) some of the tank surface preventing radiations from the screened surface. On the other-hand the tubes will improve the circulation of oil. This improves the dissipation of loss by convection. The circulation of oil is due to more effective pressure heads produced by columns of oil in tubes.

The improvement in loss dissipation by convection is equivalent to loss dissipated by 35% of tube surface area. Hence to account for this improvement in dissipation of loss by convection an additional 35 % tube area is added to actual tube surface area or the specific heat dissipation due to convection is taken as 35% more than that without tubes.

Let, The dissipating surface of the tank  =  \( S_t \)

The dissipating surface of the tubes  =  \( X S_t \)

Loss dissipated by surface of the tank by radiation and convection \( = \left( 6 + 6.5 \right) S_t = 12.5 S_t \)

Loss dissipated by tubes by convection \( = 6.5 \times \frac{135}{100} X S_t = 8.8 X S_t \)

Total loss dissipated by walls and tubes \( = 12.5 S_t + 8.8 X S_t = (12.5 + 8.8 X) S_t \)

Temperature rise in transformer with cooling tubes \( \theta = \frac{\text{Total loss}}{\text{Loss dissipated}} \)

Total loss, \( P_{\text{loss}} = P_t + P_c \)

where, \( P_t = \) Iron loss  
\( P_c = \) Copper loss

\[ \theta = \frac{P_t + P_c}{S_t(12.5 + 8.8X)} \]

(or) \[ 12.5 + 8.8X = \frac{P_t + P_c}{\theta S_t} \]

\[ \therefore X = \left( \frac{P_t + P_c}{\theta S_t} - 12.5 \right) \frac{1}{8.8} \]

Total area of cooling tubes = \( X S_t \)
On substituting for $X$ from equ

\[
\text{Total area of cooling tubes} = \frac{1}{8.8} \left[ \frac{P_1 + P_2}{\theta S_1} - 125 \right] S_1
\]

\[
= \frac{1}{8.8} \left[ \frac{P_1 + P_2}{\theta} - 125 \right] S_1
\]

Let, $l_1 = \text{Length of the tube}$

$d_1 = \text{Diameter of the tube}$

\[\therefore \text{Surface area of each tube} = \pi d_1 l_1 \]

\[\text{(surface area of a cylinder)}\]

Total number of tubes, $n_t = \frac{\text{Total area of tubes}}{\text{Area of each tube}}$

\[
n_t = \frac{1}{88 \pi d_1 l_1} \left[ \frac{P_1 + P_2}{\theta} - 125 \right] S_1
\]

The standard diameter of the cooling tubes is 50 mm and the length of the tube depends on the height of the tank. The tubes are arranged with a centre to centre spacing of 75 mm.

The inner dimensions of the transformer tank are fixed by the active dimensions of the transformer and clearances between windings and grounded parts of transformer.

\[
\text{Width of tank} \quad W_t = 2D + D_e + 2b \quad \text{(for three phase)}
\]

\[
= D + D_e + 2b \quad \text{(for single phase)}
\]

where $D = \text{distance between adjacent limbs}$,

$D_e = \text{external diameter of h.v. winding}$

and $b = \text{clearance between h.v. winding and tank}$.

Let $b = 70 \text{ mm}$

\[
\text{Length of tank} \quad L_t = D_e + 2l
\]

where $l = \text{clearance on each side between the winding and tank along the width}$.

Let $l = 90 \text{ mm}$

\[
\text{Height of transformer tank} \quad H_t = H + h
\]

Where $H = \text{Height of transformer frame}$ and

$h = \text{clearance (height) between the assembled transformer and the tank}$

$= \text{clearance at base} + \text{oil height above the assembled transformer} + \text{space for terminals and tap changing gear}$

Let $h = 50 \text{ mm} + 300 \text{ mm} + 300 \text{ mm} = 650 \text{ mm}$
EXAMPLE: 01

The tank of 1250 kVA, natural oil cooled transformer has the dimensions length, width and height as 0.65 x 1.55 x 1.85 m respectively. The full load loss = 13.1 kW, loss dissipation due to radiations = 6 W/m² °C, loss dissipation due to convection = 6.5 W/m² °C, improvement in convection due to provision of tubes = 40%, temperature rise = 40 °C, length of each tube = 1 m, diameter of tube = 50 mm. Find the number of tubes for this transformer. Neglect the top and bottom surface of the tank as regards the cooling.

**Given Data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>kVA</td>
<td>1250</td>
</tr>
<tr>
<td>l_t</td>
<td>1 m</td>
</tr>
<tr>
<td>d_t</td>
<td>50 mm</td>
</tr>
<tr>
<td>θ</td>
<td>40°C</td>
</tr>
<tr>
<td>Tank dimension</td>
<td>0.65 x 1.55 x 1.85 m</td>
</tr>
<tr>
<td>λ_conv</td>
<td>6.5 W/m² °C</td>
</tr>
<tr>
<td>λ_rad</td>
<td>6 W/m² °C</td>
</tr>
<tr>
<td>Improvement in cooling</td>
<td>40%</td>
</tr>
<tr>
<td>Full load loss</td>
<td>13.1 kW</td>
</tr>
</tbody>
</table>

**Solution**

- \[ S_t = 2 \left( L_T H_T + W_T H_T \right) = 2 \times 1.85 \times (0.65 + 1.55) = 8.14 \text{ m}^2 \]
- \[ (6 + 6.5) S_t = 12.5 S_t \]
- \[ 6.5 \times \frac{140}{100} \times X S_t = 9.1 \times X S_t \]
- \[ S_t (12.5 + 9.1 X) \]

Given that total loss, \( P_{\text{loss}} = 13.1 \text{ kW} = 13.1 \times 10^3 \text{ W} \)
Hence, \( \theta = \frac{13.1 \times 10^3}{S_t(12.5 + 9.1X)} \)

\( 12.5 + 9.1X = \frac{13.1 \times 10^3}{\theta S_t} \)

\( X = \frac{1}{9.1} \left( \frac{13.1 \times 10^3}{\theta S_t} - 12.5 \right) = \frac{1}{9.1} \left( \frac{13.1 \times 10^3}{40 \times 8.14} - 12.5 \right) = 3.0476 \)

Total area of tubes = \( XS_t = 3.0476 	imes 8.14 = 24.8075 \text{ m}^2 \)

Total number of cooling tubes = \( \frac{\text{Total Area of tubes}}{\text{Area each tube}} \)

Area of each tube = \( \pi d_t l = \pi \times 50 \times 10^{-3} \times 1 = 0.157 \text{ m}^2 \)

Total number of cooling tubes = \( \frac{24.8075}{0.157} = 158 \text{ tubes} \)

The diameter of the tube is 50 mm and the standard distance between the tubes is half of the diameter and so, let distance between tubes = 25 mm.

The width of the tank is 1550 mm. If we leave an edge spacing of 62.5 mm on either sides then we can arrange 20 tubes widthwise with a spacing of 75 mm between centres of tubes. On lengthwise we can arrange 8 tubes with same spacing as that of widthwise tubes. But one row is not sufficient to accommodate the required 158 cooling tubes. Hence three rows of cooling tubes are provided on both lengthwise and widthwise. The plan of the cooling tubes is shown in fig 4.7.1.

![Diagram of cooling tubes arrangement](image)

**Fig 4.7.1 : Plan showing the arrangement of cooling tubes for example 4.7**

**Result**

The total number of tubes provided = 160

They are arranged as 3 row on widthwise with each row consisting of 20, 19 & 20 tubes and 3 rows on lengthwise each row consisting of 8, 7 & 6 tubes.
EXAMPLE: 02

A 250 kVA, 6600/400 V, 3-phase core type transformer has a total loss of 4800 watts on full load. The transformer tank is 1.25 m in height and 1m x 0.5 m in plan. Design a suitable scheme for cooling tubes if the average temperature rise is to be limited to 35 °C. The diameter of the tube is 50 mm and are spaced 75 mm from each other. The average height of the tube is 1.05 m.

Specific heat dissipation due to radiation and convection is respectively 6 and 6.5 W/m².°c. Assume that convection is improved by 35 percent due to provision of tubes.

**Given data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>kVA</td>
<td>250</td>
</tr>
<tr>
<td>θ</td>
<td>35°C</td>
</tr>
<tr>
<td>d</td>
<td>50 mm</td>
</tr>
<tr>
<td>l</td>
<td>1.05 m</td>
</tr>
<tr>
<td>Tank dimension</td>
<td>0.5 x 1 x 1.25 m</td>
</tr>
<tr>
<td>Total power loss</td>
<td>4800 W</td>
</tr>
<tr>
<td>Distance between tube centres</td>
<td>75 mm</td>
</tr>
<tr>
<td>6600/400V; 3-phase; core type</td>
<td></td>
</tr>
</tbody>
</table>

**Solution**

\[
\begin{align*}
L_T &= \text{Length} = 0.5 \text{ m} \\
H_T &= \text{Height} = 1.25 \text{ m} \\
W_T &= \text{Width} = 1 \text{ m}
\end{align*}
\]

Heat dissipation surface of tank:

\[
S_t = \text{Total area of vertical sides} = 2(L_T H_T + W_T H_T) = 2H_T (L_T + W_T) = 2 \times 1.25 \times (0.5 + 1) = 3.75 \text{ m}^2
\]

Loss dissipated by tank walls by radiation and convection:

\[
(6 + 6.5)S_t = 12.5 S_t
\]

Loss dissipated by cooling tubes due to convection:

\[
6.5 \times \frac{135}{100} \times X S_t = 8.8 X S_t
\]

Total loss dissipated by tank and tubes:

\[
12.5 S_t + 8.8 X S_t = S_t (12.5 + 8.8 X)
\]

Temperature rise in transformer with cooling tubes:

\[
\theta = \frac{\text{Total loss}}{\text{Total loss dissipated}}
\]

Given that, Total loss, \(P_{\text{loss}} = 4800 \text{ W}\)

\[
\therefore \theta = \frac{4800}{S_t (12.5 + 8.8 X)}
\]

(or)

\[
X = \frac{1}{8.8} \left[ \frac{4800}{\theta S_t} - 12.5 \right]
\]

\[
= \frac{1}{8.8} \left[ \frac{4800}{35 \times 3.75} - 12.5 \right] = 2.7354
\]

Total area of cooling tubes = \(X S_t = 2.7354 \times 3.75 = 10.2578 \text{ m}^2\)
Area of each cooling tube \( = \pi d_1 l_1 = \pi \times 50 \times 10^{-3} \times 1.05 = 0.1649 \text{ m}^2 \)

Number of cooling tubes, \( n_t = \frac{\text{Total area of tubes}}{\text{Area of each tube}} = \frac{10.2578}{0.1649} = 62.206 \approx 62 \text{ tubes} \).

The width of the tank is 1000 mm. If we leave an edge spacing of 87.5 mm on either sides, then we can arrange 12 tubes widthwise with a spacing of 75 mm between the centres of tubes.

The length of the tank is 500 mm. If we leave an edge spacing of 100mm on either sides, then we can arrange 5 tubes lengthwise with a spacing of 75 mm between the centres of tubes.

But one row is not sufficient to accommodate the required 62 cooling tubes. Hence 2 rows of cooling tubes are provided on both lengthwise and widthwise. The plan of the cooling tubes is shown fig

Plan showing the arrangement of cooling tubes

Result

Total number of cooling tubes provided = 64

They are arranged as 2 rows on widthwise with each row consisting of 12 & 11 tubes and 2 rows on lengthwise with each row consisting of 5 & 4 tubes.

EXAMPLE: 03

A 1000 kVA, 6600/440 V, 50 Hz, 3 phase, delta/star, core type, oil immersed natural cooled (ON) transformer. The design data of the transformer is:

Distance between centres of adjacent limbs = 0.47 m, outer diameter of high voltage winding = 0.44 m, height of frame = 1.24 m.

Core loss = 3.7 kW and \( P_R \) loss = 10.5 kW.
Given data:

\[
\begin{align*}
\text{kVA} &= 1000 \\
V_p &= 6600 \text{ V} \\
V_s &= 440 \text{ V} \\
f &= 50 \text{ Hz} \\
H &= 1.24 \text{ m} \\
P_i &= 3.7 \text{ kW} \\
P_c &= 10.5 \text{ kW} \\
\theta &= 35^\circ \text{ C} \\
\lambda_{\text{con}} &= 6.5 \text{ W/m}^2 \cdot ^\circ \text{C} \\
\lambda_{\text{rad}} &= 6 \text{ W/m}^2 \cdot ^\circ \text{C} \\
D_e &= 0.44 \text{ m} \\
D &= 0.47 \text{ m} \\
\text{Improvement in cooling} &= 35 \%
\end{align*}
\]

Width of tank \( W_t = 2D + D_e + 2b \)

Assume \( b = 70 \text{ mm} = 0.07 \text{ m} \)

\[
W_t = 2 \times 0.47 + 0.44 + 2 \times 0.07 = 1.52 \text{ m}
\]

\[
W_t = 1.52 \text{ m}
\]

Length of tank \( L_t = D_e + 2L \)

Assume \( L = 90 \text{ mm} = 0.09 \text{ m} \)

\[
L_t = 0.44 + 2 \times 0.09 = 0.62 \text{ m}
\]

\[
L_t = 0.62 \text{ m}
\]

Height of tank \( H_t = H + h \)

= Height of frame + clearance between the assembled transformer and the tank

= 1.24 + (0.05 + 0.3 + 0.3)

\[
H_t = 1.89 \text{ m}
\]

Dissipating surface of the tank \( S_t = 2H_t (W_t + L_t) = 2 \times 1.89 \times (1.52 + 0.62) \)

\[
S_t = 8.0892 \text{ m}^2
\]

Loss dissipation by transformer tank = \( (6 + 6.5) S_t \)

= 12.5 \( S_t \)

Loss dissipation by tube = \( 6.5 \times \frac{135}{100} \times S_t = 8.8 \times S_t \)

Total loss dissipation by tank and tube = \( 12.5 S_t + 8.8 \times S_t \)

Total loss = \( P_i + P_c = 3.7 \times 10^3 + 10.5 \times 10^3 = 14.2 \times 10^3 \) watts

Temperature rise \( \theta = \frac{\text{Total loss}}{\text{Loss dissipated}} = \frac{P_i + P_c}{12.5 S_t + 8.8 \times S_t} \)
\[
35 = \frac{14.2 \times 10^3}{12.5 \times 8.0892 + 8.8 \times 10^3 \times 8.0892}
\]

\[x = 4.2789\]

Area of tube \(= x S_t = 4.2789 \times 8.0892 = 34.61 \text{ m}^2\)

Area of each tube \(= \pi d_t l_t = \pi \times 0.05 \times 1.4 = 0.2199 \text{ m}^2\)

Number of tubes \(n_t = \frac{\text{Total area of tube}}{\text{Area of each tube}}\)

\[= \frac{34.61}{0.2199} = 157.38 \approx 158\]

\[n_t = 158\]

Total number of cooling tubes provided = 160

They are arranged as 3 rows on widthwise with each row consisting of 20, 19 and 20 tubes and 3 rows on lengthwise with each row consisting of 8, 7 and 6 tubes.
COOLING OF TRANSFORMERS

The transformer is a static device which converts energy at one voltage level to another voltage level. During this process of energy transfer, losses occur in the windings and core of the transformer. These losses appear as heat. The heat developed in the transformer is dissipated to the surroundings. The coolants used in transformers are:

(i) air, and (ii) oil.

The transformers using air as the coolant are called dry type transformers while transformers which use oil as the coolant are called oil immersed transformers. In dry type transformers, the heat generated is conducted across the core and windings to be dissipated from the outer surfaces of windings to the surrounding air through convection.

In the case of oil immersed transformers, the heat produced inside the core and the windings is conducted across them to their surfaces. This heat is transferred by the oil to the walls of the tank through convection. Finally, the heat is transferred from the tank walls to the surrounding air by radiation and convection. It must be understood that cooling of transformers differs from that of rotating machines and presents greater problems since there are no moving parts in a transformer, that are responsible for inbuilt cooling of rotating machines.

METHODS OF COOLING OF TRANSFORMERS

There are a number of methods used for cooling of transformers. The choice of method depends upon the size, type of application and the type of conditions obtaining at the site where the transformer is installed.

The large number of methods used for dissipation of heat generated in transformers make it necessary to use a concise standard designation for them. The letter symbols used for designating these methods depend upon (i) medium of cooling used, and (ii) the type of circulation employed.

1. Medium. The cooling mediums (coolants) used for transformers along with symbols used for designating them are: (i) Air—A, (ii) Gas—G, (iii) Synthetic oil—L, (iv) Mineral oil—O, (v) Solid insulation—S, and (vi) Water—W.

2. Circulation. The circulation of the cooling medium (coolant) may be through natural means or there may be a forced circulation of the coolant. Accordingly the symbols used are:

(i) Natural—N, and (ii) Forced—F.

There are two ways of cooling a transformer:

(i) The coolant circulating inside the transformer comes in contact with the windings and cores and transfers all the heat entirely to the tank walls from where it is dissipated to the surrounding medium.

(ii) The coolant circulating inside the transformer comes in contact with windings and cores. The coolant partly transfers the heat generated to the transformer tank walls with the major portion of the heat generated inside the transformer being taken up by the coolant circulating inside the transformer, to be dissipated away later in an external heat exchanger.

The coolant circulating inside the transformer gets heated and is cooled in the heat exchanger. The heat exchanger may employ air or water in order to dissipate the heat of the coolant circulated inside the transformer.

The cooling methods are designated by symbols. Each of these letters is significant of some characteristic of the method of cooling. The cooling methods which do not employ an external heat exchanger are designated by two letters. The order in which letters are used to designate methods of cooling without external heat exchangers is:

(i) the medium in contact with the windings, and
(ii) the circulation of the coolant in contact with the windings.
These methods, therefore, are designated by two letters. The order in which letters are used designate methods of cooling using external heat exchangers is:

(i) the medium in contact with the windings,
(ii) the circulation of the coolant in contact with the windings,
(iii) the medium used in the external heat exchanger, and
(iv) the circulation of the coolant in the external heat exchanger.

The cooling methods used for dry type transformers are:

1. Air-Natural (AN). This method uses the ambient air as the cooling medium. The natural circulation of surrounding air is utilized to carry away the heat generated by natural convection. A sheet metal enclosure is used to protect the windings against mechanical damage. This method is used for small low voltage transformers. However, the development of insulating materials like glass and silicone resins class C materials which can withstand higher temperature (150°C) makes the method suitable for transformers of ratings up to 1.5 MVA. The high rating transformers are used in special applications like in mines where fire is a great hazard.

2. Air Blast (AB). Cooling by natural circulation of air becomes inadequate to dissipate heat from large transformers and hence for circulation of air (air blast) is employed in order to keep the temperature rise within limits. The forced air circulation improves the heated dissipation.

In this method, the transformer is cooled by a continuous blast of cool air forced through the cores and the windings. The air blast is produced by external fans.

The improvement in heat dissipation caused by air blast allows higher specific loadings to be used in dry type transformers. The use of higher specific loading results in lower size for the transformers. The air supply must be filtered to prevent accumulation of dust particles in the ventilating ducts.

In this method, the transformer is cooled by a continuous blast of cool air forced through the cores and the windings. The air blast is produced by external fans.

The improvement in heat dissipation caused by air blast allows higher specific loadings to be used in dry type transformers. The use of higher specific loading results in lower size for the transformers. The air supply must be filtered to prevent accumulation of dust particles in the ventilating ducts.

The cooling methods used for oil immersed transformers are:

1. Oil Natural (ON). The cooling by air is not so effective and proves insufficient for transformers of medium sizes. Oil as a coolant has two distinct advantages:

   (i) It is a better conductor of heat than air, and
   (ii) it has a high co-efficient of volume expansion with temperature. Therefore, substantial circulation is easily obtained on account of the natural “thermal head” produced due to convection so long as the cooling ducts in the cores and windings are not unduly restricted.

   Hence, almost all transformers (except for the transformers used for special applications like mines where there is a fire hazard) are oil immersed. The assembly of an oil immersed transformer is shown in Fig.

The transformer is immersed in oil and the heat generated in cores and windings is passed on to oil by conduction. Oil in contact with the heated parts rises and its place is taken by cool oil from the bottom. The heated oil transfers its heat to the tank walls from where it (heat) is taken away to the ambient air. The heated oil thereby gets cooler and falls to the bottom. Therefore, a natural thermal head is created which transfers heat from the heated parts to the tank walls from where it is dissipated to the surrounding air.

The temperature rise is given by the relationship \( \theta = Qc/S \). It is evident, that the temperature rise can be decreased and brought within limits by two means. These are:

(i) increasing \( S \), the area of heat dissipation, and
(ii) decreasing the cooling co-efficient, \( c \).
The temperature rise of a transformer is inversely proportional to $S$, its heat dissipating area. Thus if the size of the tank is increased, the dissipating area increases and hence the temperature rise decreases. Thus, in the example above if the size of the tank is increased such that its heat dissipating area is twice that which is otherwise demanded, the temperature rise of the transformer is brought down to 40°C. Therefore, this calls for an excessively large tank.

Increasing the size of tank out of proportion with the increase in transformer rating and consequent increase of dimensions, is obviously no solution for the temperature rise. The plain walled tank cannot be used beyond a particular rating. This is because if the large rating transformers used plain walled tank, the size of the tank would become enormously large. A large sized tank needs large volume of oil and hence it will result higher cost and weight of transformer. Also, beyond a particular size it would become impossible to transport the transformer (due to its large size) from the place of manufacture to the site of installation due to the limitations imposed by the gauge of the rail line or the road.

The solution to the problem of decreasing the temperature rise of large transformers lies in decreasing $c$, cooling co-efficient. The value of cooling co-efficient can be decreased by augmenting the cooling by using auxiliary means. It should be understood that as the rating of the transformer increases, the value of cooling co-efficient has to be decreased. The reduction in the value of cooling co-efficient can be brought out by the use of sophisticated methods of cooling. Therefore, as the rating of transformers increase we have to use improved methods of cooling in order to keep the temperature rise within limits. The higher rating transformers require increasingly improved methods of cooling.

Oil immersed transformers of ratings upto 30 kVA use plain walled tanks. The heat dissipating capability of transformers of ratings higher than 30 kVA is increased by providing corrugations, fins, tubes and radiator tanks. Fig. shows fins and corrugations provided on four walls. Fig. shows a transformer provided with cooling tubes. These tubes are welded to the tank walls at the top and bottom. The use of cooling tubes provides additional cooling surface but also improves the circulation of oil due to increase in thermal head.

For larger sizes of transformers, radiator tanks with fins or corrugations are employed. Fig. shows a transformer provided with external radiators.

It is clear, that in the method described above the oil in the transformer is circulated on account of natural thermal head. The oil takes away the heat from inside the transformer to outside. The oil is cooled in tubes or external radiators by natural circulation of air. Therefore, the methods described above can be termed as ONAN (oil natural and air natural).
Transformers up to a capacity of about 5 MVA or a loss of up to 50 kW use tanks with tubes. The tubes are usually round and are 55 mm in diameter and are arranged in one to three rows. Elliptical tubes are also used.

2. Oil Natural Air Forced (ONAF). In this method the oil circulating under natural head transfers heat to the tank walls. The transformer tank is made hollow and air is blown through the hollow space to cool the transformer. The heat removed from the inner tank walls can be increased to five or six times that dissipated by natural means and therefore very large transformers can be cooled by this method. However, the normal way of cooling the transformers by air blast is to use radiator banks of corrugated or elliptical tubes separated from the transformer tank and cooled by air blast produced by fans.

3. Oil Natural Water Forced (ONWF). In this method, copper cooling coils are mounted above the transformer core but below the surface of oil. Water is circulated through the cooling coils to cool the transformer.

This method proves to be cheap where a natural water head is already available.

The method has, however, the serious disadvantage that it employs a cooling system which carries water inside the oil tank. Since the water is at higher head than oil, therefore, in case of leakage water in the cooling tubes will enter the transformer tank contaminating oil and reducing its dielectric strength.

Since heat passes three times as rapidly from copper cooling tubes to water as from oil to copper tubes, the tubes are provided with fans to increase conduction of heat from oil to tubes. The water inlet and outlet pipes are lagged in order to prevent the moisture in the ambient air from condensing on the pipes and getting into the oil.

4. Forced Circulation of Oil (OF). In large transformers the natural circulation of oil is insufficient for cooling the transformer and forced circulation is employed. Oil is circulated by a motor driven pump from the top of a transformer tank to an external cooling plant (heat exchanger or refrigerator) where the oil is cooled. The cold oil enters the transformer at the bottom of the tank.

The method of cooling oil in the heat exchanger depends upon the condition obtained at the site. The methods of cooling transformers by forced circulation of oil are classified accordingly as:

- 47 -
(i) Oil Forced Air Natural (OFAN). In this method oil is circulated through the transformer with the help of a pump and cooled in a heat exchanger by natural circulation of air. This method is not commonly used. However, this method proves very useful where the coolers have to be well removed from the transformer.

(ii) Oil Forced Air Forced (OFAF). The method is depicted in Fig. The oil is cooled in external heat exchangers using air blast produced by fans. It is interesting to note that the oil pump and fans may not be used all the time. At low loads, the losses are small and therefore natural circulation of oil with an ONAN condition may be sufficient to cool the transformer. At higher loads, the pump and the fans may be switched on by temperature sensing elements. Therefore mixed cooling conditions are used, the transformer working with ONAN conditions upto 50% of rating and OFAF conditions at higher loads. This arrangement results in higher efficiency for the system.

(iii) Oil Forced Water Forced (OFWF). In this method the heated oil is cooled in a water heat exchanger. The pressure of oil is kept higher than that of water and therefore any leakage that occurs is from oil to water. Also there are no condensation problems. At sites, where the cooling water has a considerable head, it is usual to employ cascaded heat exchangers i.e. oil/water and water/water with the intermediate water circuit being at a low pressure. This cooling method is suitable for banks of transformers, but from the system reliability considerations not more than, say, three tanks should be connected in one cooling pump circuit. The advantages of OFWF method over ONWF are that the transformer is smaller and the transformer tank does not have to contain cooling coils carrying water.

The use of water as a coolant is common at generating stations, particularly hydro-electric stations, where large supply of water is available.

Transformers with a capacity of upto 10 MVA have a cooling radiator system with natural cooling.

The forced oil and air circulation (OFAF) method is the usual one for transformers of capacities 30 MVA upwards.

As stated earlier, the forced oil and water (OFWF) is used for transformers designed for hydro-electric plants.
TWO MARKS QUESTION AND ANSWERS

01 What are the salient features of distribution transformer?
- The distribution transformers will have low iron loss and higher value of copper loss.
- The capacity of transformers will be upto 500 kVA
- The transformers will have plain walled tanks or provided with cooling tubes or radiators.
- The leakage reactance and regulation will be low.

02 What are the salient features of distribution transformer?
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03 What is the advantage in shell type transformer over core type transformer. 
In shell type transformers the coils are well supported on all the sides and so they can withstand higher mechanical stresses developed during short circuit conditions. Also the leakage reactance will be less in shell type transformers.

04 In transformers, why the low voltage winding is placed near the core? 
The winding & core are both made of metals and so an insulation have to be placed inbetween them. The thickness of insulation depends on the voltage rating of the winding. In order to reduce the insulation requirement the low voltage winding is placed near the core.

05 Write down the output equation of 1-phase and 3-phase transformer
The equation which relates the rated kVA output of the transformer to core and window area is called output equation.
Output kVA of single phase transformer, \( Q = 2.22 f B_m A_i K_w A_w \delta \times 10^{-3} \) 
Output kVA of three phase transformer, \( Q = 3.33 f B_m A_i K_w A_w \delta \times 10^{-3} \)

06 What do you meant by stacking factor (or iron space factor)? What is its usual value?
In transformers, the core is made of laminations and the laminations are insulated from each other by a thin coating of varnish. Hence when the laminations are stacked to form the core, the actual iron area will be less than the core area. The ratio of iron area and total core are is called stacking factor.
Stacking factor, \( S_f = \frac{\text{Area of cross-section of iron in the core}}{\text{Area of cross-section of the core including the insulation area}} \)

The usual value of stacking factor is 0.9.

How will you select the emf per turn of a transformer?
The equation of emf per turn in terms of kVA rating, flux, frequency and ampere-turn is given by,

\[
E_t = K \sqrt{Q}
\]

where, \( K = (4.44 f \frac{\phi_m}{AT} \times 10^3)^{1/2} \)

For a given kVA rating the emf per turn is directly proportional to \( K \). The value of \( K \) depends on the type, service condition and method of construction of transformer. The value of \( K \) for different types of transformers are listed in the following table.

<table>
<thead>
<tr>
<th>Transformer type</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single phase shell type</td>
<td>1.0 to 1.2</td>
</tr>
<tr>
<td>Single phase core type</td>
<td>0.75 to 0.85</td>
</tr>
<tr>
<td>Three phase shell type</td>
<td>1.3</td>
</tr>
<tr>
<td>Three phase core type, distribution transformer</td>
<td>0.45</td>
</tr>
<tr>
<td>Three phase core type, power transformer</td>
<td>0.6 to 0.7</td>
</tr>
</tbody>
</table>

List the different types of windings used in core type transformers.

The different types of windings employed in core type transformers are
- Cylindrical winding
- Helical winding
- Double helical winding
- Multi-layer helical winding
- Cross-over winding
- Disc and continuous disc winding
- Aluminium foil winding

List the different methods of cooling of transformers.

The different methods of cooling of transformers are,
- Air Natural (AN)
- Air Blast (AB)
- Oil Natural (ON)
- Oil Natural Air Forced (ONAF)
- Oil Natural Water Forced (ONWF)
- Oil Forced (OF)
- Oil Forced Air Natural (OFAN)
- Oil Forced Air Forced (OFAF)
- Oil Forced Water Forced (OFWF)

What are the factors to be considered for selecting the cooling method of a transformer?
The choice of cooling method depends on kVA rating of transformer, size, application and the site condition where it will be installed.
10 Why transformer oil is used as a cooling medium?
   When transformer oil is used as a coolant the heat dissipation by convection is 10 times more than the convection due to air. Hence the transformer oil is used as coolant.
   Specific heat dissipation by convection due to air = \(8 \text{ W/m}^2\cdot{^\circ}\text{C}\)
   Specific heat dissipation by convection due to oil = 80 to 100 \(\text{W/m}^2\cdot{^\circ}\text{C}\)

11 How the heat dissipation is improved by the provision of cooling tubes?
   The cooling tubes will improve the circulation of oil. The circulation of oil is due to more effective pressure heads produced by columns of oil in tubes. The improvement in cooling is accounted by taking the specific heat dissipation due to convection as 35\% more than that without tubes.

12 In mines applications transformers with oil cooling should not be used why?
   The oil used for transformer cooling is inflammable (i.e., can easily set on fire), hence leakage of cooling oil may create fire accidents in mines. Therefore oil cooled transformers are not used in mines.

13 What is breather?
   The breather is a device fitted in transformer for breathing. In small oil cooled transformers some air-gap is provided between the oil level and tank top surface. When the oil is heated, it expands and air is expelled out of transformer to the atmosphere through breather. When the oil is cooled, it shrinks and air is drawn from the atmosphere into the transformer through breather. This action of transformer is called breathing.

14 What is conservator?
   A conservator is a small cylindrical drum fitted just above the transformer main tank. It is used to allow the expansion and contraction of oil without contact with surrounding atmosphere.
   When conservator is fitted in a transformer, the tank is fully filled with oil and the conservator is half filled with oil.

15 What are the advantages of using higher flux density in the core?
   1. Reduction in core and yoke section for same output.
   2. Reduction in mean length of LV and HV turns, resulting in saving of copper material, reduced over all size and weight of transformer.

16 List various disadvantages of using higher flux density in design of core
   1. Increased magnetizing current and iron losses,
   2. Saturation of magnetic material,
   3. Lower efficiency, because of higher no load losses, higher temperature rise of transformer.

17 Name a few insulating materials that are used in transformers.
   Press board, cable paper, varnished silk, transformer oil, porcelain, insulating varnish
18 State the disadvantages of 3 phase transformers over single phase transformer.

1. It is more difficult to transport a 3 phase transformer as the weight per unit is more.
2. In the event of a fault in any phase of a 3 phase transformer, the fault is transferred to the other two phases. Therefore, the whole unit needs replacement.

19 State the advantages of cold-rolled grain oriented (CRGO) steel used for magnetic circuits of transformer.

1. Magnetic induction is maximum and the loop of BH curve is large.
2. Core loss during no load operation of transformer is low.
3. Reactive power input at no-load operation of the transformer is low.
4. Magnetostriction is low.
5. Good mechanical properties.

20 Write the relation between core area and weight of iron and copper for a single phase transformer.

Area of core \( A_i = \frac{Q}{\sqrt{\frac{G_i}{fB_m G_c}}} \)

21 Why is the core of a transformer laminated?

Eddy current loss can be reduced by core of thin laminations. Hence the core of transformer core is laminated.

22 Explain the concern while doing the design of distribution transformer.

(i) The distribution transformers are designed to have low iron loss and higher copper loss.
(ii) The distribution transformers are designed to have the maximum efficiency at a load much lower than full load (about 50 percent).
(iii) The distribution transformers should have a good voltage regulation and therefore they should be designed for a small value of leakage reactance.

23 Explain the significance of the ratio \( r = \frac{\Phi_m}{AT} \) in the design of transformer.

Transformers may be designed to make one of the following quantities as minimum. (i) Total volume (ii) Total weight (iii) Total cost (iv) Total losses.

All these quantities vary with the ratio \( r = \frac{\Phi_m}{AT} \). If we choose a high value of \( r \), the flux becomes larger and consequently a large core cross-section is needed which results in higher volume, weight and cost of iron and also gives a higher iron loss. On the other hand, owing to decrease in the value of \( AT \) the volume, weight and cost of copper required decrease and also the copper losses decreases. Thus we conclude that the value of \( r \) is controlling factor for the above mentioned quantities at the time of design of transformer.
24 List out the advantages and disadvantages of stepped cores.

Advantages:
For same area of cross-section the stepped cores will have lesser diameter of circumscribing circle than square cores. This results in reduction in length of mean turn of the winding with consequent reduction in both cost of copper and copper loss.

Disadvantages:
With large number of steps a large number of different sizes of laminations have to be used. This results in higher labour chargers for shearing and assembling different types of laminations.

25 What an empirical relation which is normally used for the estimation of window space factor in the design of transformer.

\[ K = \frac{8}{30 + kV} \] for transformer of about 20kVA rating

\[ K = \frac{10}{30 + kV} \] for transformer of rating between 50 to 200 kVA rating

\[ K = \frac{12}{30 + kV} \] for transformer of about 1000 kVA rating

where \( kV \) is the voltage of h.v winding in kilo-volt.

26 State the reason for preferring circular coils in comparison to rectangular coil in transformer windings.

The excessive leakage fluxes produced during short circuit and over loads develop severe mechanical stresses on the coils. On circular coils these forces are radial and there is no tendency for the coil to change its shape. But on rectangular coils the force are perpendicular to the conductors and tends to deform the coil in circular form.

27 Top and bottom surfaces of the transformer tank are not considered for the design of cooling tubes for transformer. Why?

The area of top and bottom surfaces of the transformer tank are not considered for the design of cooling tubes, as it has very little cooling effect.

28 Define window space factor used in the design of a transformer.

The window space factor is defined as the ratio of copper area in the window to total window area.

\[ K_{w} = \frac{\text{Conductor area in window}}{\text{total area of window}} = \frac{A_{c}}{A_{w}} \]
29 What are the advantages of three phase transformers over single phase transformers.

1. A 3 phase transformer is lighter, occupies lesser space, cheaper and more efficient than a bank of single phase transformers.
2. The installation and operational costs are smaller for 3 phase units.

30 The voltage/turn of a 500 kVA, 11 kV/415 V, Delta/Star, 3 phase transformer is 8.7 V. Calculate the number of turns per phase of LV and HV windings.

Phase voltage of LV winding = \( \frac{415}{\sqrt{3}} = 239.6 \text{ V} \) (\( \therefore \) Star connected)

Phase voltage of HV winding = 11,000 V (\( \therefore \) Delta connected)

\[
\text{Number of turns} = \frac{\text{Phase voltage of LV winding}}{\text{Emf per turn}} = \frac{239.6}{8.7} = 27.54 \approx 28 \text{ turn}
\]

\[
\text{Number of turns} = \frac{\text{No. of turns of LV winding} \times \text{Phase voltage ratio}}{11,000} = 28 \times \frac{239.6}{11,000} = 12855 \approx 1286 \text{ turns}
\]

31 Draw the cruciform section of the transformer core and give the optimum dimensions in terms of circumscribing circle diameter ‘d’.

The optimum dimensions of a & b in terms of d are

\[
a = 0.85 \text{ d} \\
b = 0.53 \text{ d}
\]

32 Distinguish between shell type and core type transformer.

<table>
<thead>
<tr>
<th>Core type</th>
<th>Shell type</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Has low mechanical strength due to non-bracing of windings.</td>
<td>2. High mechanical strength.</td>
</tr>
<tr>
<td>3. Reduction of leakage reactance is not easily possible.</td>
<td>3. Reduction of leakage reactance is highly possible.</td>
</tr>
<tr>
<td>4. The assembly can be easily dismantled for repair work.</td>
<td>4. It cannot be easily dismantled for repair work.</td>
</tr>
<tr>
<td>5. Better heat dissipation from windings.</td>
<td>5. Heat is not easily dissipated from windings since it is surrounded by core.</td>
</tr>
<tr>
<td>6. Has longer mean length of core and shorter mean length of coil turn. Hence best suited for EHV (Extra High Voltage) requirements.</td>
<td>6. It is not suitable for EHV (Extra High Voltage) requirements.</td>
</tr>
</tbody>
</table>
33 State different losses in a transformer.

The losses in a transformer are of two types namely

(i) Core or iron losses
The core losses consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux.

\[
\text{Hysteresis loss} = K_h f B_m^{1.6} \text{ Watts/m}^3
\]

\[
\text{Eddy current loss} = K_e f^2 B_m^2 t^2 \text{ watts/m}^3
\]

(ii) Copper losses
These losses occur in both the primary and secondary windings due to their ohmic resistance.

\[
\text{Total Cu losses, } P_c = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} \text{ or } I_2^2 R_{02}
\]