5.1 INTRODUCTION

A permanent magnet synchronous motor is also called as brushless permanent magnet sine wave motor. A sine wave motor has a

1. Sinusoidal or quasi-sinusoidal distribution of magnetic flux in the air gap.
2. Sinusoidal or quasi-sinusoidal current wave forms.
3. Quasi-sinusoidal distribution of stator conductors (i.e.) short-pitched and distributed or concentric stator windings.

The quasi sinusoidal distribution of magnetic flux around the air gap is achieved by tapering the magnet thickness at the pole edges and by using a shorter magnet pole arc typically 120°.

The quasi sinusoidal current wave forms are achieved through the use of PWM inverters and this may be current regulated to produce the best possible approximation to a pure sine wave. The use of short pitched distributed or concentric winding is exactly the same as in ac motors.

5.2 CONSTRUCTION AND PRINCIPLE OF OPERATION

Permanent magnet synchronous machines generally have same operating and performance characteristics as synchronous machines. A permanent magnet machine can have a configuration almost identical to that of the conventional synchronous machines with absence of slip rings and a field winding.

Construction

Fig. 5.1 shows a cross section of simple permanent magnet synchronous machines. It consists of the stationary member of the machine called stator. Stator laminations for axial air gap machines are often formed by winding continuous strips of soft steel. Various parts of the laminations are the teeth slots which contain the armature windings. Yoke completes the magnetic path. Lamination thickness depends upon the frequency of the armature source voltage and cost.

Armature windings are generally double layer (two coil side per slot) and lap wound. Individual coils are connected together to form phasor groups. Phasor groups are connected together in series/parallel combinations to form star, delta, two phase (or) single windings.

AC windings are generally short pitched to reduce harmonic voltage generated in the windings.
Coils, phase groups and phases must be insulated from each other in the end-turn regions and the required dielectric strength of the insulation will depend upon the voltage ratings of the machines.

![Fig. 5.1 structure of the stator and rotor](image)

In a permanent magnet machines the air gap serves an role in that its length largely determines the operating point of the permanent magnet in the no-load operating condition of the machines. Also longer air gaps reduce machines windage losses.

The permanent magnets form the poles equivalent to the wound field pole of conventional synchronous machines. Permanent magnet poles are inherently —salient‖ and there is no equivalent to the cylindrical rotor pole configurations used in many conventional synchronous machines.

Many permanent magnet synchronous machines may be cylindrical or —smooth rotor‖ physically but electrically the magnet is still equivalent to a salient pole structure. Some of the PMSM rotors have the permanent magnets directly facing the air gap as in fig. 5.2.

Rotor yoke is the magnetic portion of the rotor to provide a return path for the permanent magnets and also provide structural support. The yoke is often a part of the pole structure.

![Fig. 5.2 PMSM rotor](image)

Damper winding is the typical cage arrangement of conducting bars, similar to induction motor rotor bars and to damper bars used on many other types of synchronous machines. It is not
essential for all permanent magnet synchronous machines applications, but is found in most machines used in power applications.

The main purpose is to dampen the oscillations about synchronous speed, but the bars are also used to start synchronous motors in many applications.

The design and assembly of damper bars in permanent magnet machines are similar to the other types of synchronous machines.

Synchronous machines are classified according to their rotor configuration. There are four general types of rotors in permanent magnet synchronous machines. They are

1. Peripheral rotor
2. Interior rotor
3. Claw pole or Lundell rotor.
4. Transverse rotor.

**Peripheral rotor**

The permanent magnets are located on the rotor periphery and permanent magnet flux is radial.

**Interior rotor**

The permanent magnets are located on the interior of the rotor and flux is generally radial.

**Claw pole or Lundell**

The permanent magnets are generally disc shaped and magnetized axially. Long soft iron extensions emanate axially from periphery of the discs like claws or Lundell poles. There is a set of equally spaced claws on each disc which alternate with each other forming alternate north and south poles.

**Transverse rotor**

In this type the permanent magnets are generally between soft iron poles and the permanent magnet flux is circumferential. In this soft iron poles at as damper bars. Magnetically this configuration is similar to a reluctance machine rotor, since the permeability of the permanent magnet is very low, almost the same as that of a non-magnetic material. Therefore, reluctance torque as well as torque resulting from the permanent magnet flux is developed.

Thus BLPM sine waves (SNW) motor is construction wise the same as that of BLPM square wave (SQW) motor. The armature winding and the shape of the permanent magnet are so designed that flux density distribution of the air gap is sinusoidal(i.e.) .The magnetic field setup by the permanent magnet in the air gap is sinusoidal
5.3 EMF EQUATION OF BLPM SINE WAVE MOTOR

5.3.1 Flux density distribution
Flux density can be expressed as $B = \Phi B \sin\theta$ or $B \cos p\theta$ or $B \sin(p\theta + \alpha)$ or $B \cos(p\theta + \alpha),$ $2p = p,$ (i.e.) $p$-no of pole pairs depending upon the position of the reference axis as shown in fig 5.3.

![Fig 5.3 flux density distribution](image)

Consider a full pitched single turn armature coil as shown in fig 5.4. Let the rotor be revolving with a uniform angular velocity of $\omega_m$ mech. rad/sec.
At time $t = 0,$ let the axis of the single turn coil be along the polar axis.

![Diagram of flux density distribution](image)

Consider a small strip of $d\theta$ mech. radians at a position $\theta$ from the reference.
Flux density at the strip $B = \Phi B \sin p\theta$
Incremental flux in the strip $d\theta = B X$ area swept by the conductor
$d\theta = \Phi B \sin p\theta X lr d\theta$
$B lr d\theta$ weber
Where
$L$ – Length of the armature in m
$r$ – Radians of the armature
$d\theta = \Phi B \sin p\theta X lr d\theta$
$= B \Phi lr \sin p\theta X d\theta$
Flux enclosed by the coil after lapses of $t$ sec is

$$\phi = lr \sin P\theta d\theta$$

----------(5.1)
\[ \phi = (2 \mathbb{B} \square \text{lr}/p) \cos p\theta \omega_{mt} \]

### 5.3.2. EMF Equation of an ideal BLPM sine wave motor

As per Faraday's law of electromagnetic induction, emf induction in the single turn coil.

\[ e = -N \frac{d \phi}{dt} \]

- \( d\phi /dt \) as \( N=1 \)
- \( e = - \frac{d\phi}{dt} \left( \frac{(2 \mathbb{B} \square \text{lr}/p)}{p \omega_{mt}} \right) \cos p\theta \omega_{mt} \)

\[ e = 2 \mathbb{B} \square \text{lr} \omega_m \sin p \omega_{mt} \quad \ldots \ldots (5.2) \]

Let the armature winding be such that all turns of the phase are concentrated full pitched and located with respect to pole axis in the same manner.

Let \( T_{ph} \) be the number of turns connected in series per phase. Then the algebraic addition of the emfs of the individual turns gives the emf induced per phase as all the emf are equal and in phase.

\[ e_{ph} = \text{lr} \omega_m \sin p \omega_{mt} T_{ph} \quad \ldots \ldots (5.3) \]

\[ e_{ph} = \mathbb{E}_{ph} \sin \omega_{et} \]

\[ \mathbb{E}_{ph} = 2 \mathbb{B} \square \text{lr} \omega_m T_{ph} \omega_m \]

\[ \mathbb{E}_{ph} = \text{rms value of the phase emf} \]

\[ = \mathbb{E}_{ph} / \sqrt{2} \]

\[ = \sqrt{2} \mathbb{B} \square \text{lr} \omega_m T_{ph} \omega_m \]

\[ \omega_m = \omega_e / \rho \]

\( \phi_m \) – sinusoidal distributed flux / pole

\[ \phi = B_{av} \tau l \]

\[ = B_{av} \times (2\pi r / 2p) \times l \]

Average value of flux density for sinewave = \( 2/\pi \)

\[ = (2/\pi) B_{av} \]

\[ \phi_m = (2 / \pi) \mathbb{B} \square X (\pi r / P). l \]
\[ \phi_m = \left( \frac{2 B \square r_1}{P} \right) \]

\[ B \square r_1 = \left( \frac{P \phi_m}{2} \right) \]

\[ E_{ph} = \sqrt{2} B \square l r \omega_m T_{ph}, \text{volt} \]

Sub equ

\[ E_{ph} = \sqrt{2} \left( \frac{P \phi_m}{2} \right) \omega_m T_{ph} \]

\[ = \sqrt{2} \left( \frac{P \phi_m}{2} \right) \left( \frac{\omega}{p} \right) T_{ph} \]

\[ = \sqrt{2} \left( \frac{P \phi_m}{2} \right) \left( \frac{2\pi f}{p} \right) T_{ph} \]

\[ E_{ph} = 4.44 f \phi_m T_{ph}, \text{Volt} \]

5.3.3 EMF equation of practical BLPM sine wave motor

In a practical BLPM sine wave motor at the time of design it is taken care to have the flux density is sinusoidal distributed and rotor rotates with uniform angular velocity. However armature winding consists of short chorde coils properly distributed over a set of slot.

These aspect reduce the magnitude of \( E_{ph} \) of an ideal winding by a factor \( K_{w1} \) which is known as the winding factor the fundamental component of flux.

\[ K_{w1} = K_{s1} K_{p1} K_{b1} \]

\[ K_{s1} = \text{slew factor} \]

\[ K_{s1} = \frac{\sin \sigma/2}{\sigma/2} \]

\[ K_{s1} = 1 \text{ (slightly less than 1) } \]

\[ \sigma - \text{Skew angle in elec. Radians.} \]

\[ K_{p1} = \text{pitch factor (or) short chording factor} \]

\[ = \sin m \frac{\pi}{2} \text{ or} \]

\[ \cos \frac{\rho}{2} \text{ Where } m = \]

coil span/pole pitch

\[ \pi \text{ elec rad} \]

\[ \pi/\rho \text{ mech. Rad} \]
\[ K_{p1} = \sin \frac{m\pi}{2} \text{ or } \cos \frac{p}{2} \]

\[ m \pi \text{ is elec rad, } \frac{m\pi}{p} \text{ mech. Rad. } \]

\[ K_{b1} = \text{Distribution factor or width factor} \]

\[ K_{b1} = \frac{e^{i\pi}}{q \sin^2} \]

Where \( v \) = slot angle in elec. Radians

\[ = \frac{2\pi}{n_s} \pi \frac{p}{s}, \quad n_s = \text{no. of slots (total)} \]

\[ q = \text{slots/pole/phase for 60° phase spread} \]

\[ = \text{slots/pair of poles/phase} \]

\[ K_{b1} < 1; \quad K_{p1} < 1; \quad K_{s1} < 1 \]

Therefore \( K_{w1} = K_{p1} K_{b1} K_{s1} < 1 \) (winding factor)

Thus rms value of the per phase emf is

\[ E_{ph} = 4.44 f T_{ph} K_{w1} \text{ volts.} \quad \ldots \ldots (5.9) \]

**5.4. TORQUE EQUATION OF BLPM SINE WAVE MOTOR**

**5.4.1. Ampere conductor density distribution**

Let the fig. 5.5 shows the ampere conductor density distribution in the airgap due to the current carrying armature winding be sinusoidal distributed in the airgap space.

\[ A = A^\sin \theta \]

Where \( A = \text{ampere conductor density} \)

\[ = \text{ampere conductor/degree} \]
Consider a strip of d $\theta$ at an angle $\theta$ from the reference axis.

Ampere conductor in the strip $d \theta = A \, d\theta$

$$= A^\wedge \sin P \theta \, d\theta$$ .......(5.10)

Ampere conductor per pole $= \int_0^P A^\wedge \sin P \theta \, d\theta$ .......(5.11)

$$= -A^\wedge \left[ \frac{\cos P \theta}{P} \right]$$

$$= -\frac{A^\wedge}{P} \left[ \cos \frac{\pi}{P} - \cos 0 \right]$$

$$= \frac{2}{P} A$$

Let $T_{ph}$ be the number of full pitched turns per phase.

Let $i$ be the current

$i \, T_{ph}$ be the total ampere turns which is assumed to be $\theta$ sine distributed.

Total ampere conductors [sine distributed] $= 2i \, T_{ph}$

Sine distributed ampere conductors/pole $= \frac{2i}{2P} \tau_{ph}$

Equating eqn. 6.30 and eqn. 6.32

$$\frac{2}{P} A = \frac{2i}{2P} \tau_{ph}$$

$$A^\wedge = \frac{i T_{ph}}{2}$$ .......(5.12)

**5.4.2. Torque equation of an ideal BLPM sine wave motor:**

Let the ampere conductor distribution of ideal BLPM sine wave motor be given by $A = A^\wedge \sin P \theta$

Let the flux density distribution set up by the rotor permanent magnet be also sinusoidal.

Let the axis of armature ampere conductor distribution be displaced from the axis of the flux density distribution by an angle $(\frac{\pi}{2} - \alpha)$ as shown in fig 5.6

$[ B = B^\wedge \sin \left( P \theta + \frac{\pi}{2} - \alpha \right) ]$ .......(5.13)
\[ B = B^\wedge \sin \left( \frac{P \theta - \alpha}{2} \right) \]
\[ = B^\wedge \cos \left( P \theta - \alpha \right) \]
\[ B = B^\wedge \cos \]

Consider a small strip of width \( d \theta \) at an angle \( \theta \) from the reference axis.

Flux density at the strip \( B = B^\wedge \cos(p\Theta - \alpha) \)

Ampere conductors in the strip \( = Ad\theta \)

\[ = A \sin P\Theta \ d\Theta \]

Force experienced by the armature conductors in the strip \( d\Theta = BIAd\Theta \)

\[ dF = B^\wedge \cos(P\Theta - \alpha)1.A\cdot B\cdot1.A\cdot \sin P\Theta .d\Theta \]

\[ dF = A^\wedge B^\wedge 1 \sin P\Theta \cos(P\Theta - \alpha) \ d\Theta . \]

Let \( r \) be the radial distance of the conductors from the axis of the shaft.

Torque experienced by the ampere conductors of the strip=dF*r

\[ dT = AB \ r l \ \sin P\Theta \ \cos(P\Theta - \alpha) \ D\theta \ N \cdot m \]

Torque experienced by the ampere conductors/pole \( T/Pole = \int_0^{\pi/p} dT \)

\[ T = \int_0^{\pi} A B \ r l \ \sin P \theta \ \cos \left( P \theta - \alpha \right) \ d \theta \]

\[ = A B r l / 2 \ \int_0^{\pi/p} \left( \sin P \theta + P \theta \alpha + \sin \alpha \right) \ d \theta \]

\[ = A B r l / 2 \left[ \frac{\cos(\alpha)}{\cos(\cdot)} - \frac{P \theta - \alpha}{2} \right] + \theta \sin \alpha \]

\[ = A B r l / 2 - \left[ \frac{\cos \alpha}{\cos \alpha} - \frac{\pi}{\sin \alpha} \right] \]

\[ T = A B r l / 2. - \ N - m \]

The total torque experienced by all the armature conductors

\[ = 2P \times \text{torque/pole} \]

\[ = 2P \times - \]

\[ T = \pi A B r l \ \sin \alpha \ N \cdot m \ldots \]
As the armature conductors are located in stator of the BLPM SNW motor, the rotor experiences an equal and opposite torque.

\[ T = -\pi A B r l \sin \alpha \]

\[ = \pi A B r l \sin \beta \text{ where } \beta = -\alpha \] 

\[ ......(5.19) \]

B is known as power angle or torque angle. \( T = \pi A B r l \sin \beta \) in an ideal motor.

Consider the case of an armature winding which has three phases. Further the winding consists of short chording coils and the coils of a phase group are distributed. The 3 phase armature winding carries a balanced 3 phase ac current which are sinusoidally varying. The various phase windings are ph a, ph b and ph c.

The axis of phase winding are displaced by \( 2\pi/3p \) mechanical radians or \( 2\pi/3 \) elec. Radians. The current in the winding are also balanced. An armature winding is said to be balanced if all the three phase winding are exactly identical in all respects but there axes are mutually displaced by \( 2\pi/3p \) mech radians apart.

A three phase armature current is said to be balanced when the 3 phase currents are exactly equal but mutually displaced in phase by 120 degree.

Let

\[ i_a = l m \cos \omega t \]

\[ i_b = l m \cos \left( \omega t - \frac{2\pi}{3} \right) = \sqrt{2} l \cos \left( \omega t - 2\pi/3 \right) \] 

\[ i_c = \phi m \left( \omega t + \frac{2\pi}{3} \right) = \sqrt{2} l \cos \left( \omega t - 4\pi/3 \right) \] 

\[ ......(5.20) \]

\[ ......(5.21) \]

\[ ......(5.22) \]

When the 3 phase ac current passes through the 3 phase balanced winding it sets up an armature mmf in the air gap.

Space distribution of the fundamental component of armature ampere conductors can be written as.

\[ = \cos P \theta \] 

\[ ......(5.23) \]
\[ f_a = F_m \]
\[ f_b = F_m \cos (\theta - 2\pi/3) \]  
\[ f_c = F_m \cos (\theta - 4\pi/3) \]  

\[ \text{5.4.3 Torque developed in a practical BLPM SNW motor:} \]

- Ampere turn distribution of a phase winding consisting of full pitched coil is rectangular of amplitude \( I \times T_{ph} \). But the fundamental component of this distribution is the fundamental component of this distribution is \( 4/\pi I \times T_{ph} \).

- In a practical motor, the armature turns are short chorded and distributed. Further, they may be accommodated in skewed slots. In such a case, for getting the fundamental component of ampere turns distribution, the turns per phase is modified as \( K_{w1} T_{ph} \) where \( K_{w1} \) is the winding factor which is equal to \( K_{s1} K_{p1} K_{d1} \)

\[ K_{s1} = \text{Skew factor} = \sin \sigma/2; \sigma = \text{skew angle in elec. rad.} \]

\[ K_{p1} = \sin \frac{m\pi}{2} \quad m\pi = \text{coil span in elec. Rad} \]

\[ K_{d} = \text{distribution factor} = \frac{\sin q\nu/2}{q \sin \nu} \quad v = \text{slot angle in electrical rad, q = slot per pole for 60degree phase spread.} \]

Fundamental component of ampere turns per phase of a practical one

\[ = \frac{4}{\pi} I \times T_{ph} K_{w1} \]  

\[ \text{when a balanced sinusoidally varying 3 phase ac current pass through a balanced 3 phase winding it can be shown that the total sinusoidally distributed ampere turns is equal to} \]

\[ = \frac{4}{\pi} \cdot \frac{3}{2} \times \sqrt{2} \times \text{Iph} \times K_{w1} \times T_{ph} \]  

\[ \text{4. The amplitude of the ampere conductor density distribution is shown is equal to the total sinusoidally distributed ampere turns divided by 2.} \]

\[ \text{Therefore } \bar{A} \text{ in a practical 3 phase motor } = \frac{3/2 \times \sqrt{2}}{\text{Iph} \times K_{w1} \times T_{ph}} \]

\[ \text{Electromagnetic torque developed in a practical BLPL SNW motor} \]

\[ = \pi \times A \times B \times r \times l \sin \beta \]  

\[ = \pi \times B \times r \times l \sin \beta \]
\[
\left[ 3 \sqrt{\frac{2}{\pi}} l_{ph} K w_{1} T_{ph} \right]
= 3(\sqrt{2}Kw_{1} T_{ph} B r l) l_{ph} \sin \beta
= 3 \frac{E_{ph}}{\omega_{m}} l_{ph} \sin \beta
\]

\[
i_a T_{ph} = I_{max} \cos \omega t \cos \theta
\]

\[
i_b T_{ph} = I_{max} \cos \left( \omega t - \frac{2\pi}{3} \right) \cos \left( \theta - \frac{2\pi}{3} \right)
\]

\[
i_c T_{ph} = I_{max} \cos \left( \omega t - \frac{4\pi}{3} \right) \cos \left( \theta - \frac{4\pi}{3} \right)
\]

\[
i T_{ph} = i_a T_{ph} + i_b T_{ph} + i_c T_{ph}
\]

\[
= I_{max} \left( \frac{\cos(\omega t + \theta) + \cos(\omega t - \theta)}{2} \right) + I_{max} \left( \frac{\cos(\omega t + \theta - 4\pi/3) + \cos(\omega t - \theta)}{2} \right)
\]

\[
= \frac{1}{2} I_{max} \cdot 3 \cos(\omega t - \theta) + \frac{1}{2} I_{max} \cdot \cos(\omega t + \theta) \cos 2\theta + \sin(\omega t + \theta) \sin 2\theta + \cos(\omega t + \theta) \cos 2\theta + \sin(\omega t + \theta) \sin 2\theta
\]

\[
= \frac{3}{2} I_{max} \cos(\omega t - \theta) + \frac{1}{2} I_{max} \cdot \cos(\omega t + \theta) - 0.866 \sin(\omega t + \theta) - 0.5 \cos(\omega t + \theta) + 0.866 \sin(\omega t + \theta)
\]

\[
= \frac{1}{2} I_{max} T_{ph} \cos(\omega t - \theta)
\]

Properties of ‘A’ (Ampere conductor density);

- Ampere conductor density is sinusoidally distributed in space with amplitude \( \hat{A} \). This distribution has 2p poles (i.e) same as the rotor permanent magnetic field.
- The ampere conductor distribution revolves in air gap with uniform angular velocity \( \omega_{m} \) rad/sec or \( \omega_{elec} \) rad/sec (Ns rpm). This is the same speed as that of rotor magnetic field.
- The direction of rotation of armature ampere conductor distribution is same as that of rotor. This is achieved by suitably triggering the electronic circuit from the signals obtained from rotor position sensor.
- 4. The relative angular velocity between sine distributed permanent magnetic field and sine distributed armature ampere conductor density field is 0. Under such condition it has been shown an electromagnetic torque is developed whose magnitude is proportional to \( \sin \beta \).

\( \beta \)-torque angle or power angle.

Angle between the axes of the two fields is \( \pi/2 - \alpha \) and \( \beta = -\alpha \)

Torque developed by the motor = \( 3E_{ph} I_{ph} \sin \beta / \omega_{m} \) N-m

Where \( \omega_{m} \)-angular velocity in rad/sec.
\[ \omega_m = \frac{2\pi N_s}{60} \]  where \( N_s \) is in rpm

\[ T = \frac{60}{2\pi N_s} (3E_{ph}I_{ph}\sin\beta) \]

\[ = 3E_{ph}I_{ph}\sin\beta \text{ syn.watts.} \]

1 syn.watt = \( \frac{60}{2\pi N_s} \) N-m

It is a machine dependent conversion factor

### .5 PHASOR DIAGRAM OF A BRUSHLESS PM SNW OR BLPB SYNCHRONOUS MOTOR:

Consider a BLPM SNW motor, the stator carries a balanced 3υ winding. This winding is connected to a dc supply through an electronic commutator whose switching action is influenced by the signal obtained from the rotor position sensor.

Under steady state operating condition, the voltage available at the input terminals of the armature winding is assumed to be sinusoidally varying three phase balanced voltage. The electronic commutator acts as an ideal inverter whose frequency is influenced by the rotor speed. Under this condition a revolving magnetic field is set up in the air gap which is sinusoidally distributed in space, having a number of poles is equal to the rotor. It rotates in air gap in the same direction as that of rotor and a speed equal to the speed of the rotor

Rotor carries a permanent magnet. Its flux density is sine distributed. It also revolved in the air gap with as particular speed

It is assumed that the motor acts as a balanced 3υ system. Therefore it is sufficient to draw the phasor diagram for only one phase. The armature winding circuit is influenced by the following emfs.

1. \( V \) - supply voltage per phase across each winding of the armature.
   The magnitude of this voltage depends upon dc voltage and switching techniques adopted.

2. \( E_f \) - emf induced in the armature winding per phase due to sinusoidally varying permanent magnetic field flux. Magnitude of
   \[ E_f = 4.44\nu_{mf}K_wI_{ph}\] = \( I_{E_f} \)
   As per Faradays law of electromagnetic induction, this emf lags behind \( \nu_{mf} \)-permanent magnet flux enclosed by armature phase winding by 90°.

3. \( E_a \) - emf induced in the armature phase winding due to the flux \( \nu_a \) set up by resultant armature mmf \( \nu \times I_a \)
1. \( E_a = I_a X_a \) where \( X_a = 4.44 f K K w_1 T_{ph} \)

This lags behind \( V_a \) by 90° or in other words \( E_a \) lags behind \( I_a \) by 90°.

Therefore \( E_a = -jX_a I_a \)

4. \( E_{al} \) - emf induced in the same armature winding due to armature leakage flux.

\[
|E_{al}| = 4.44 f \phi_{al} K_{w_1} T_{ph}
\]

\( \phi_{al} \) is the leakage flux and is directly proportional to \( f \).

Therefore \( |E_{al}| = 4.44 f (K_a l_{al} K_{w_1} T_{ph}) \)

\[
|E_{al}| = I_a X_{al}
\]

Where \( X_{al} = 4.44 f K a l_{al} K_{w_1} T_{ph} \) in the leakage inductance, \( E_{al} \) lags behind \( \phi_{al} \)

Or \( I_a \) by 90°

Therefore \( E_{al} = -jI_a X_{al} \)

**Voltage equation:**

The Basic voltage equation of the armature circuit is

\[
V' + \dot{E}_f + \dot{E}_{al} = \dot{I}_a R_a
\]

Where \( R_a \) is the resistance per phase of the armature winding.

\[
V' + \dot{E}_f - j \dot{I}_a X_a - j \dot{I}_a X_{al} = \dot{I}_a R_a
\]

\[
V' + \dot{E}_f - j \dot{I}_a (X_a + X_{al}) = \dot{I}_a R_a
\]

\[
V' + \dot{E}_f - j \dot{I}_a X_s = \dot{I}_a R_a
\]

Where \( X_s = X_a + X_{l} \)

\( X_s \) is known as synchronous reactance per phase or fictitious reactance.

\[
V = (\text{-}E_f) + I_a (R_a + jX_s)
\]

\[
V' = \dot{E}_q + \dot{I}_a Z_s
\]

Where \( Z_s \) is the synchronous impedance.

Let \( E_q \) be the reference phasor. Let it be represented by \( OA \).
Let $I$ be the current phasor. OB represents $I$.

$E_f$ be the emf induced in the armature winding by permanent magnet flux $= -Eq$

OC represents $E_f$

![Fig 5.7 phasor diagram of BLPM sine wave motor](image)

$\phi_{mj}$ be the mutual flux set up by the permanent magnet, but linked by the armature winding.

$E_f$ lags behind $\phi_{mj}$

AF represents $I_a R_a$

FG represents $I_a X_s$; FG is perpendicular to $I$ phasor

OG represents $V$

Angle between the $I$ and $\phi_{mj}$ is $\beta$ the torque or power angle.

Power input $= 3VI$

$$= 3 (Eq + I_a R_a + j I X_s).I$$

$$= 3 Eq I + 3 R_a + O \quad \text{........(5.37)}$$

$3Eq I$ – electromagnetic power transferred as mechanical power.

$3 R_a$ – copper loss.

Mechanical power developed $= 3 Eq I \quad \text{........(5.38)}$

$$= 3 Eq I \cos(90 - \beta)$$

$$= 3 Eq I \sin \beta$$

$$= 3 E_f I \sin \beta \quad \text{........(5.39)}$$

The motor operates at $N_s$ rpm or $120f/2p$ rpm.
Therefore electromagnetic torque developed is 

\[ = P/ \]

\[ = 3E_q I \sin \beta \]  \hspace{1cm} \ldots (5.40)

**Fig 5.8 Phasor Diagram of BLPM sine wave motor with \( \phi_d \) or \( \phi_{mf} \) as reference axis**

Further the current \( I \) phasor is resolved into two components

\[ I = I_d + j I_q \]  \hspace{1cm} \ldots (5.42)

Therefore \( V = E_q + I_d R_a + j I_q X_s \) \hspace{1cm} \ldots (5.41)

\[ V = (V_r + j V_i q) \]

From the above drawn phasor.

\[ V = (I_d \phi_d - I_q X_s) + j (E_q + I_q \phi_m + I_d X_s) \]

I can also be represented as a complex quantity

\[ I = I_d + j I_q \]

Power input = Re (3VI*) \hspace{1cm} I* - conjugate

\[ = \text{Re}(3((I_d \phi_d - I_q X_s) + j (E_q + I_q \phi_m + I_d X_s)) ((I_d - j I_q))) \]
Torque developed for a salient pole machine is given by

\[ T = \frac{3p}{\omega_m} \left[ E_q I_q \left( X_d - X_q \right) I_d I_q \right] \]

In case of surface – magnet motors, the reluctance torque becomes zero.

Therefore, torque developed = \( \frac{3p}{\omega_m} E_q I_q N \cdot m \)

Or = \( \frac{3p}{\omega} E_q I_q N \cdot m \)
At a given speed, $E_q$ is fixed as it is proportional to speed. Then torque is proportional to q-axis current $I_q$.

The linear relationship between torque and current simplifies the controller design and makes the dynamic performance more regular and predictable. The same property is shared by the square wave motor and the permanent commutator motor.

In the phasor diagram shown in fig. 5.10.

![Fig 5.9 Phasor Diagram neglecting the effect of resistance](image)

Neglecting the effect of resistance, the basic voltage equation of BLPMSNW motor

(i.e.,) $\dot{V} = E_q^* + j \dot{X}$

As the effect of resistance is neglected

$$\frac{\dot{V}}{jX_s} = \frac{E_q}{jX_s} + \dot{j} \quad \ldots \ldots (5.44)$$

$$\dot{j} = \frac{\dot{V} - E_q}{jX_s} \quad \ldots \ldots (5.45)$$

For a particular frequency of operation the phasor diagram can be drawn as shown in figure.
5.6. PERMISSIBLE TORQUE-SPEED CHARACTERISTICS

The torque-speed characteristics of BLPM sine wave motor is shown in fig. 5.10

![Torque-Speed Characteristics Diagram]

**Fig 5.10 torque-speed characteristics of BLPM sine wave (SNW) motor.**

For a given $V_c$ and $I_c$ (i.e) maximum permissible voltage and maximum permissible current, maximum torque remains constant from a low frequency to $f_c$ (i.e) corner frequency.

Any further increase in frequency decreases the maximum torque. At $f = f_D$(i.e.) $f_{max}$ the torque Developed is zero. Shaded pole represents the permissible region of operation in torque speed characteristics.

**Effect of over speed**

In the torque speed characteristics, if the speed is increased beyond the point D, there is a risk of over current because the back emf $E_q$ continues to increase while the terminal voltage remains constant. The current is then almost a pure reactive current flowing from the motor back to the supply. There is a small $q$ axis current and a small torque because of losses in the motor and in the converter. The power flow is thus reversed. This mode of operation is possible only if the motor ‘over runs’ the converter or is driven by an external load or prime mover.

In such a case the reactive current is limited only by the synchronous reactance. As the speed increase further, it approaches the short circuit current $\frac{E_q}{X_s}$ which is many times larger than the normal current rating of the motor winding or the converter. This current may be sufficient to demagnetize the magnets particularly if their temperature is high. Current is rectified by the freewheeling diodes in the converter and there is a additional risk due to over voltage on the dc side of the converter, especially if a filter capacitor and ac line rectifiers are used to supply the dc. But this condition is unusual, even though in the system design the possibility should be assessed.
Solution

An effective solution is to use an over speed relay to short circuit the 3υ winding in a 3υ resistor or a short circuit to produce a braking torque without actually releasing the converter.

5.7. VECTOR CONTROL OF BLPM SNW MOTOR

Electromagnetic torque in any electrical machine is developed due to the interaction of current carrying armature conductors with the air gap flux. Consider a two machine whose armature conductor currents and air gap flux are as shown in fig. 5.12. Here the flux is in quadrature with the armature mmf axis.

Each and every armature conductor experiences a force which contributes the torque. The torque contributed by various armature conductors have the same direction even through their magnitude may vary. It is observed that the steady state and dynamic (behaviors) performance of a most of such an arrangement are better.

Consider a second case wherein the armature conductor current distribution and air gap flux distribution are as shown in fig. 6.26. In this case the angle between the axis of the air gap flux and the armature mmf axis is different from 90° elec.

In this case also each and every armature conductor experiences a force and contributes to the torque. But in this case the direction of the torque experienced by the conductors is not the same. Since conduction develops torque in one direction while the others develop in the opposite direction. As a result, the resultant torque gets reduced; consequently it is observed that both the steady state and dynamic performance of such a motor is poorer.

For a BLPM motor to have better steady state and dynamic performance, it is essential that the armature mmf axis and the axis of PM are to be in quadrature for all operating condition.
5.7.1. Principle of vector control

BLPM SNW motor is usually employed for variable speed applications. For this we keep V/f constant and vary V and f to get the desired speed and torque.

From the theory of BLPM SNW motor it is known that as the speed is varied from a very low value up to the corner frequency, the desired operating point of current is such that \( I_d = 0 \) and \( I \) is along the \( q \)-axis. Such a condition can be achieved by suitably controlling the voltage by PWM technique after adjusting the frequency to a desired value.

When the frequency is more than the corner frequency it is not possible to make \( I_d = 0 \), due to the voltage constraints. In such a case a better operating point for current is obtained with minimum \( I_d \) value after satisfying the voltage constraints. Controlling BLPM SNW motor taking into consideration the above mentioned aspects is known as “vector Control” of BLPM SNW motor.

5.7.2. Schematic Diagram of Vector Control

The schematic block diagram of vector control is as shown in figure 5.13 Knowing the value of the desired torque and speed and also the parameters and the voltage to which the motor is subjected to, it is possible to complete the values of \( i_d \text{.ref} \) and \( i_q \text{.ref} \) for the desired dynamic and steady state performance.

![Schematic diagram of vector control](image)

RPS – Rotor position sensor, TG – Tachogenerator

**Fig.5.13 Schematic diagram of vector control**

The reference values of \( i_d \) and \( i_q \) are transformed into reference values of currents namely \( i_a \text{.ref}, i_b \text{.ref} \) and \( i_c \text{.ref} \). These currents are compared with the actual currents and the error values actuate the triggering circuitry which is also influenced by the rotor position sensor and speed. Thus the vector control of BLPM SNW motor is achieved.
5.8 SELF CONTROL OF PMSM

As the rotor speed changes the armature supply frequency is also change proportionally so that the armature field always moves (rotates) at the same speed as the rotor. The armature and rotor field move in synchronism for all operating points. Here accurate tracking of speed by frequency is realized with the help of rotor position sensor.

When the rotor makes certain predetermined angle with the axis of the armature phases the firing pulses to the converter feeding the motor is also change. The switches are fired at a frequency proportional to the motor speed. Thus the frequency of the voltage induced in the armature is proportional to the speed.

Self-control ensures that for all operating points the armature and rotor fields move exactly at the same speed. The torque angle is adjusted electronically hence there is an additional controllable parameter passing greater control of the motor behavior by changing the firing of the semi-conductor switches of an inverter.

The torque angle is said electronically hence the fundamental component of phase A needs $\Phi_f/\beta$, it lies along the direct axis that rotates at a synchronous speed. The switches must be triggered by phase A current component when $\Phi_f$ axis is $\beta$ electrical degrees behind the phase A axis. This is achieved by firing the switch when direct axis is $\delta+\beta$ behind axis of A as show shown in fig.

Self-control is applicable to all variable frequency converters, the frequency being determined by machine.

![Fig 5.14 Schematic diagram of self-control](image)

At high power levels the most common power converter configuration is the current fed DC link converter which is shown in fig. 5.14.
5.8.1 Inner current and outer speed loop

The phase controlled thyristor rectifier on the supply side of the DC link has the current regulating loop and operate as a control current source. The regulated DC current is delivered to the DC link inductor to the thyristor of load commutator inverter which supplies line current to the synchronous motor.

The inverter gating signals are under the control of shaft-position sensor giving a commutator less dc motor with armature current controlled. The thyristor of these inverters utilize load commutation because of the generated emf appearing at the armature. It is ensured by the over excitation of synchronous motor, so that it operates at leading power factor hence it reduces commutating circuitry, low losses and is applicable to power levels of several megawatts.

The shaft position is sensed by the position sensor. The shaft speed is obtained by converting the position information. This speed is compared with the reference speed signal which provides the speed error. This is the current reference signal for the linear current loop.

This reference current is compared with the sensed dc link current which provides control signals for the rectifier thyristor. The sensed shaft position is used as gating signal for inverter thyristor.

5.8.2 Commutation at low speed

Load commutation is ensured only at high speeds. Whereas at low speeds the emf generated is not sufficient for load commutation. The inverter can be commutated by supplying pulsating on and off dc link current. This technique produces large pulsating torque but this is not suitable for drives which require smooth torque at low speed.

The DC link current is pulsed by phase shifting the gate signal of the supply side converter from rectification to inversion and back again. When the current is zero the motor side converter is switched to a new conduction period and supply side converter is then turned on. Time required for the motor current to fall to zero can be significantly shortened by placing a shunt thyristor in parallel with a DC link inductor. When the current zero is needed the line side converter is phased back to inversion and the auxiliary thyristor is gated.

The DC link inductor is then short circuited and its current can supply freely without affecting the motor. When the line side converter is turned on the auxiliary thyristor is quickly blocked. This method of interruption of the motor current reduces the effect of pulsating torque.

5.8.3 Four Quadrant Operations

The drive characteristics are similar to those of a conventional DC motor drive. Motor speed can be increased to a certain base speed corresponding to the maximum voltage from the supply. Further, increase in speed is obtained by reducing the field current to give a field weakening region of operation.
Regenerative braking is accomplished by shifting the gate signal, so that machine side inverter acts as a rectifier and supply side rectifier as an inverter, hence the power is return to the ac utility network. The direction of rotation

Of the motor is also reversible by alternating the gate sequence of the motor side converter. Thus four quadrant operations are achieved, without additional circuitry.

5.9 MICROPROCESSOR BASED CONTROL OF PMSE

Fig.5.15 Microprocessor Based Control of PMSM

Fig 5.15 shows the block diagram of microprocessor based permanent magnet synchronous motor drive.

The advent of microprocessor has raised interest in digital control of power converter systems and electronics motor drives since the microprocessor provides a flexible and low cost alternative to the conventional method.

For permanent magnet synchronous motor drive systems, microprocessor control offers several interesting features principally improved performance and reliability, versatility of the controller, reduced components and reduced development and manufacturing cost. In the block diagram of the microprocessor controller PMSM shown in fig 5.15, the permanent magnet synchronous motor is fed from a current source d.c link converter system, which consists of a SCR inverter through rectifier and which is operated from three phase a.c supply lines, and its gating signals are provided by digitally controlled firing circuit.

The optical encoder which is composed of a coded disk attached to the motor shaft and four optical sensors, providing rotor speed and position signals. The inverter triggering pulses are synchronized to the rotor position reference signals with a delay angle determined by an 8-bit control input. The inverter SCR’s are naturally commutated by the machines voltages during normal conditions. The speed signals, which is a train of pulses of frequency, proportional to the motor speed, is fed to a programmable counter used for speed sensing.
The stator current is detected by current sensor and amplified by optically isolated amplifier. The output signals are multiplexed and converted to digital form by a high speed analog to digital converter.

The main functions of the microprocessor are monitoring and control of the system variables for the purpose of obtaining desired drive features. It can also perform various auxiliary tasks such as protection, diagnosis and display. In normal operation, commands are fetched from the input-output terminals, and system variables (the dc link current, the rotor position and speed) are sensed and fed to the CPU. After processing, the microprocessor issues control signal to the input rectifier, then the machine inverter, so as to provide the programmed drive characteristics.