2.1 INTRODUCTION

It is an electrodynamics and electromagnetic equipment.

These motors are also referred to as step motors or stepping motors.

On account of its unusual construction, operation and characteristics it is difficult to define a stepper motor.

A stepper motor is brushless dc motor whose rotor rotates in discrete angular displacements when its stator windings are energized in a programmed manner. Rotation occurs because of magnetic interaction between rotor poles and poles of the sequentially energized winding. The rotor has no electrical windings, but has salient and magnetic/or magnetized poles.

The stepper motor is a digital actuator whose input is in the form of digital signals and whose output is in the form of discrete angular rotation. The angular rotation is dependent on the number of input pulses the motor is suitable for controlling the position by controlling the number of input pulses. Thus they are identically suited for open position and speed control.

Applications:

- Printers
- Graph plotters
- Tape driver
- Disk Drives
- Machine Tools
- X-Y Recorders
- Robotics space Vehicle
- IC Fabrication and Electric Watches

2.2 CLASSIFICATION OF STEPPER MOTORS

As construction is concerned stepper motors may be divided into two major groups.

1. **Without Permanent Magnet**
   - (a) Single Stack
   - (b) Multi Stack

2. **With Permanent Magnet**
   - (a) Claw Pole Motor
   - (b) Hybrid Motors
2.3 SINGLE STACK VARIABLE RELUCTANCE STEPPER MOTOR

2.3.1 Construction:

The VR stepper motor characterized by the fact there is no permanent magnet either on the rotor or the stator. The construction of a 3-phase VR stepper motor with 6 poles on the stator and 4-pole on the rotor as shown.

![Diagram of Single Stack Variable Reluctance Stepper Motor](image)

The Stator is made up of silicon steel stampings with inward projected even or odd number of poles or teeth. Each and every stator poles carries a field coil an exciting coil. In case of even number of poles the exciting coils of opposite poles are connected in series. The two coils are connected such that their MMF gets added. the combination of two coils is known as phase winding.

The rotor is also made up of silicon steel stampings with outward projected poles and it does not have any electrical windings. The number of rotor poles should be different from that of stators in order to have self-starting capability and bi direction. The width of rotor teeth should be same as stator teeth. Solid silicon steel rotors are extensively employed. Both the stator and rotor materials must have lowering a high magnetic flux to pass through them even if a low magneto motive force is applied.

2.3.2 Electrical Connection

Electrical connection of VR stepper as shown fig. Coil A and A' are connected in series to form a phase winding. This phase winding is connected to a DC source with the help of semiconductor switch S1. Similarly B and B' and C and C' are connected to the same source through semiconductor switches S2 and S3 respectively. The motor has 3 –phases a, b and c.

- a phase consist of A and A' Coils
- b phase consist of B and B' Coils
- c phase consist of C and C' Coils
2.3.3 Principle of Operation

It works on the principle of variable reluctance. The principle of operation of VR stepper motor explained by referring fig.

(a). Mode 1: One phase ON or full step operation

In this mode of operation of stepper motor only one phase is energized at any time. If current is applied to the coils of phase \(_a’\) (or) phase \(_a^*\) is excited, the reluctance torque causes the rotor to run until aligns with the axis of phase \(a\). The axis of rotor poles 1 and 3 are in alignment with the axis of stator poles \(_A’\) and \(_A^{*}\). Then angle \(\theta = 0^\circ\) the magnetic reluctance is minimized and this state provides a rest or equilibrium position to the rotor and rotor cannot move until phase \(_a’\) is energized.

Next phase \(b\) is energized by turning on the semiconductor switch \(S2\) and phase \(_a’\) is de-energized by turning off \(S1\). Then the rotor poles 1 and 3 and 2 and 4 experience torques in opposite direction. When the rotor and stator teeth are out of alignment in the excited phase the magnetic reluctance is large. The torque experienced by 1 and 3 are in clockwise direction and that of 2 and 4 is in counter clockwise direction. The latter is more than the former. As a result the rotor makes an angular displacement of 30° in counterclockwise direction so that \(B\) and \(B’\) and 2 and 4 in alignment. The phases are excited in sequence \(a, b\) and \(c\) the rotor turns with a step of 30° in counterclockwise direction. The direction of rotation can be reversed by reversing the switching sequence in which are energized and is independent of the direction of currents through the phase winding.

![Fig 2.3 step motions as switching sequence process in a three phase VR motor](image)

The truth table for mode I operation in counter and clockwise directions are given in the table
(b). Mode II: Two Phase on Mode

In this mode two stator phases are excited simultaneously. When phases a and b are energized together, the rotor experiences torque from both phases and comes to rest in a point midway between the two adjacent full step position. If the phases b and c are excited, the rotor occupies a position such that angle between AA’ axis of stator and 1-3 axis of rotor is equal to 45°. To reverse the direction of rotation switching sequence is changed a and b, a and c etc. The main advantage of this type of operation is that torque developed by the stepper motor is more than that due to single phase ON mode of operation.

The truth table for mode II operation in counter clockwise and clockwise directions is given in a table.

### Table 2.3: Counter Clockwise Rotation (CCW)

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>*</td>
<td>-</td>
<td>15° AB</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>*</td>
<td>45° BC</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>-</td>
<td>75° CA</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>-</td>
<td>105° AB</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>*</td>
<td>135° BC</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>-</td>
<td>165° CA</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>-</td>
<td>195° AB</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>*</td>
<td>225° BC</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>-</td>
<td>255° CA</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>-</td>
<td>285° AB</td>
</tr>
</tbody>
</table>
Table 2.4: Clockwise Rotation (CW) (C)

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>-</td>
<td>*</td>
<td>-</td>
<td>15°</td>
</tr>
<tr>
<td>CB</td>
<td>-</td>
<td>*</td>
<td>*</td>
<td>45°</td>
</tr>
<tr>
<td>BA</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>75°</td>
</tr>
<tr>
<td>AC</td>
<td>-</td>
<td>*</td>
<td>-</td>
<td>105°</td>
</tr>
<tr>
<td>CB</td>
<td>-</td>
<td>*</td>
<td>*</td>
<td>135°</td>
</tr>
<tr>
<td>BA</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>165°</td>
</tr>
<tr>
<td>AC</td>
<td>-</td>
<td>*</td>
<td>-</td>
<td>195°</td>
</tr>
<tr>
<td>CB</td>
<td>-</td>
<td>*</td>
<td>*</td>
<td>225°</td>
</tr>
<tr>
<td>BA</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>255°</td>
</tr>
<tr>
<td>AC</td>
<td></td>
<td></td>
<td></td>
<td>285°</td>
</tr>
</tbody>
</table>
In this type of mode of operation on phase is ON for some duration and two phases are ON during some other duration. The step angle can be reduced from 30° to 15° by exciting phase sequence a, a+b, b,b+c, c etc. The technique of shifting excitation from one phase to another from a to b with an intermediate step of a+b is known as half step and is used to realize smaller steps. Continuous half stepping produces smoother shaft rotation.

The truth table for mode III operation in counter and clockwise directions are given in the table:

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>-</td>
<td>-</td>
<td>0°</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>-</td>
<td>15°</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>-</td>
<td>30°</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>*</td>
<td>45°</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>*</td>
<td>60°</td>
</tr>
<tr>
<td>*</td>
<td>-</td>
<td>*</td>
<td>75°</td>
</tr>
<tr>
<td>*</td>
<td>-</td>
<td>-</td>
<td>90°</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>-</td>
<td>105°</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>-</td>
<td>120°</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>*</td>
<td>135°</td>
</tr>
<tr>
<td>-</td>
<td>*</td>
<td>-</td>
<td>150°</td>
</tr>
<tr>
<td>*</td>
<td>-</td>
<td>*</td>
<td>165°</td>
</tr>
</tbody>
</table>

2.4 MICRO STEPPING CONTROL OF STEPPING MOTOR

Stepping motor is a digital actuator which moves in steps of θs in response to input pulses. Such incremental motion results in the following limitations of the stepper motor:

Limited resolution

As θs is the smallest angle through which the stepper motor can move, this has an effect on position accuracy of incremental servo system employing stepper motors because the stepper motor cannot position the load to an accuracy finer than θs.

Mid frequency Resonance

A phenomenon in which the motor torque suddenly drops to a low value at certain pulse frequencies as in fig.

![Fig 2.4 Mid frequency Resonance](image)

A new principle known as micro stepping control has been developed with a view of...
overcoming the above limitation. It enables the stepping motor to move through a tiny micro step of size $\Delta \theta_s \ll \theta_s$ full step angle in response to input pulses.

### 2.5. MULTISTACK VARIABLE RELUCTANCE STEPPER MOTOR

These are used to obtain smaller step sizes, typically in the range of 2° to 15°. Although three stacks are common, a multistack motor may employ as many as seven stacks. This type is also known as the cascade type. A cutaway view of a three stack motor is shown in fig. 2.6.

A multistack (or m-stack) variable reluctance stepper motor can be considered to be made up of ‘m’ identical single stack variable reluctance motors with their rotors mounted on a single shaft. The stators and rotors have the same number of poles (or teeth) and therefore same pole (tooth) pitch. For a m0stack motor, the stator poles (or teeth) in all m stacks are aligned, but the rotor poles (teeth) are displaced by 1/m of the pole pitch angle from one another. All the stator pole windings in a given stack are exited simultaneously and, therefore the stator winding of each stack forms one phase. Thus the motor has the same number of phases as number of stacks.

Figure 2.7 shows the cross section of a three stack (3-phase) motor parallel to the shaft. In each stack, stator and rotors have 12 poles (teeth). For a 12 pole rotor, pole pitch is 30° and therefore, the rotor poles (teeth) are displaced from each other by 1/3rd of the pole pitch or 10°. The stator teeth in each stack are aligned. When the phase winding A is excited rotor teeth of stack A are aligned with the stator teeth as shown in fig. 2.8.

When phase A is de-energized and phase B is excited the rotor teeth of stack B are aligned with stator teeth. The new alignment is made by the rotor movement of 10° in the anticlockwise
direction. Thus the motor moves one step (equal to $\frac{1}{2}$ pole pitch) due to change of excitation from stack A to stack B.

Next phase B is de-energized and phase C is excited. The rotor moves by another step $\frac{1}{3}$rd of pole pitch in the anticlockwise direction. Another change of excitation from stack C to stack A will once more align the stator and rotor teeth in stack A. However during this process ($A \rightarrow B \rightarrow C \rightarrow A$) the rotor has moved one rotor tooth pitch.

Let $N_r$ be the number of rotor teeth and $m$ the number of stacks or phases, then

Tooth pitch $T_p = \frac{360}{N_r}$ \hspace{1cm} \text{................. (2.1)}

Step Angle $\alpha = \frac{360^\circ}{mN_r}$ \hspace{1cm} \text{................. (2.2)}
2.6. Hybrid stepper motor

Principle of operation

Most widely used hybrid motor is the two phase type as shown in fig2.11. This model has four poles and operates on one phase on excitation.

![Fig 2.9 Cross-section of a two phase hybrid motor](image)

The coil in pole 1 and that in pole 3 are connected in series consisting of phase A, and pole 2 and 4 are for phase B. Fig 2.12 shows the process of rotor journey as the winding currents are switched in one phase on excitation.

![Fig 2.10 One-phase on operation of a two-phase hybrid motor.](image)

The poles of phase A are excited the teeth of pole 1 attract some of the rotors north poles, while the teeth of pole 3 align with rotor's south poles. Current is then switched to phase B. The rotor will travel a quarter tooth pitch so that tooth alignment takes place in 2 and 4.

Next current is switched back to phase A but in opposite polarity to before, the rotor will make another quarter tooth journey. The tooth alignment occurs in opposite magnetic polarity to state 1. When current is switched to phase B in opposite polarity (4) Occurs as a result of quarter tooth pitch journey.

The structures of two phase motor considered in fig.2.11 will not produce force in a symmetrical manner with respect to the axis. The motor having 8 poles in the stator shown in fig2.13 considered as the structure in which torque is generated at a symmetrical position on the surface.
2.9. THEORY OF TORQUE PREDICTION

According to Faraday's laws of electromagnetic induction:

Theory of Torque Predictions in stepping motors:

A qualitative approach is not always suitable for the treatment of stepper motors in terms of circuitry parameters. So, a theory based on the magnetic energy and co-energy is to be studied to analyze the mechanism of torque production - dynamic approach.

The analysis is started with ideal case in which the rotor and stator cores have infinite permeability, and the next case will be the cores are subjected to the magnetic saturation.

Case II: Infinitely permeable cores:

A current I flows through coil of N turns to yield magnetic flux, and a force f acts on iron piece in x-direction. Here, the iron piece is the teeth of the rotor. The electromagnet corresponds to a pair of teeth of stator.

Let us consider, \( B_g \) - Magnetic flux density in the air gap.
According to amperes law,
\[ \oint H \cdot dl = nI \]  \hspace{1cm} (1)
\[ \oint H \cdot dl = H_g (g/A) + H_i (l) \]  \hspace{1cm} (2)
\[ = H_g (g) + H_i (l) \]

where, \( H_g \) - Magnetic field intensity in airgap.
\( H_i \) - Magnetic field intensity in the cores.
\( l \) - Total magnetic path in the cores.

When permeability of cores is extremely large, \( H_i (l) = 0 \).

So, \( \oint H \cdot dl = H_g (g) \)  \hspace{1cm} (3)
\[ H_g \cdot g = nI \]
\[ H_g = \frac{nI}{g} \]  \hspace{1cm} (4)

We know that, \( B_g = \mu_0 H_g \Rightarrow H_g = \frac{B_g}{\mu_0} \)

Substitute \( H_g \) in (4),
\[ B_g = \frac{\mu_0 nI}{g} \]  \hspace{1cm} (5)

where, \( \mu_0 \) - Permeability in the gap length.

Let \( w_o \) - Transverse length of iron piece.
\( x \) - Distance by which rotor tooth & iron piece overlap

Overlapped area = \( x \times w_o \).
\[ B_g = \frac{\phi}{A} = \frac{\phi}{x \times w_o} \]
\[ \phi = \frac{\mu_0 nI \times w_o}{g} \]  \hspace{1cm} (6)
Flux linkages, \( \Phi = n \Phi \) — (7)

Substitute (6) in (7),
\[
\Phi = \frac{x \omega \Phi_0 n^2 I}{g}
\] — (8)

Let us assume that there is an incremental displacement \( dx \) at time \( \Delta t \),
\[
\Delta \Phi = \frac{\omega \Phi_0 n^2 I dx}{g} — (9)
\]

Emf, \( e = -\frac{\Delta \Phi}{\Delta t} = -\frac{\omega \Phi_0 n^2 I}{g} \cdot \frac{dx}{\Delta t} — (10)\)

Work done, \( \Delta P_t = I|e| \Delta t \)
\[
\Delta P_t = \frac{\omega \Phi_0 n^2 I^2}{g} \cdot \frac{dx}{\Delta t}
\] — (11)

 Coil resistance is zero,
\[
\Delta P_t = Bq^2 \cdot \frac{g \omega dx}{\mu_0}
\] — (12)

Work done by the source is converted partly to mechanical work and the rest in increasing the magnetic field energy in the gaps.
\[
\Delta W_m = \frac{1}{2} \Delta P_t
\]
\[
\Delta W_m = \frac{1}{2} Bq^2 \cdot \frac{g \omega dx}{\mu_0} — (13)\]
AE \Phi_i is converted into magnetic field energy and other
half of \AE \Phi_i is converted into the mechanical work. Since the
mechanical work is the force 'f' multiplied by the displacement 'x'.

Comparing (15) \times (16),

\[ f = \frac{1}{2} \frac{B_0^2}{\mu_0} g w \] —— (15)

Substitute (15) to (16),

\[ f = \frac{1}{2} \frac{\mu_0 n^2 I^2}{g^2 \mu_0} g w \]

\[ f = \frac{1}{2} \frac{\omega \mu_0 n^2 I^2}{g} \] —— (16)

Magnetic energy \( W_m = \int f x \)

\[ W_m = \frac{1}{2} \frac{B_0^2}{\mu_0} q x w \]

\[ W_m = \frac{1}{2} \frac{\mu_0 n^2 I^2 x w}{g} \] —— (17)

Compare (15) \times (17),

\[ f = \left[ \frac{\delta W_m}{\delta x} \right] I = \text{constant} \quad (\text{in rigorous form}) \]

\[ f = \left[ \frac{d W_m}{d x} \right] I = \text{constant} \]

\[ f = -\left[ \frac{d W_m}{d x} \right] \phi = \text{constant} \quad (\text{coil resistance is not zero}) \]
Case (ii) - Constant permeabilities:

In the infinitely permeable cores, the magnetic field appears only in the gaps. When cores are of finite permeability, the magnetic energy appears also in the cores and spaces other than the gaps.

\[ \Phi = LI \]  \hspace{1cm} (18)

where \( \Phi \) = flux linkages
\( L \) = coil inductance

The magnetic energy,

\[ W_m = \frac{1}{2} LI^2 \]

\[ E_{mf}, e = -\frac{\Delta \Phi}{\Delta t} = -\Delta (LI) \]

\[ e = -I \frac{\Delta L}{\Delta t} \]  \hspace{1cm} (19)

Work \( \Delta P_i \) can be expressed as,

\[ \Delta P_i = I I_{c} \Delta t \]

\[ = I \left(-I \frac{\Delta L}{\Delta t}\right) \Delta t \]

\[ \Delta P_i = I^2 \Delta L \]  \hspace{1cm} (20)

Increase in the magnetic energy \( \Delta W_m \),

\[ \Delta W_m = \frac{1}{2} \Delta P_i \]

\[ \Delta W_m = \frac{1}{2} I^2 \Delta L \]  \hspace{1cm} (21)
Mechanical work, $\Delta P_0 = f \Delta x$ — (22)

Comparing (21) $\times$ (22),

$\Delta x = \frac{1}{2} I^2 \Delta L$

$f = \frac{1}{2} I^2 \frac{\Delta L}{\Delta x}$ — (22)

In the above procedure, it was assumed that the coil resistance was zero and the power supply was a current source.

**Effects of mutual induction:**

Number of drive schemes for stepping motors are

1. One phase on drive.
2. Two phase on drive.
3. Three phase on drive.

In a drive scheme, other than one phase on drive, it is desirable that the mutual inductance is minimum. When mutual inductance is not negligible, the torque in terms of linear theory is derived by the following procedure.
mutual Induction,

\[ e = - \frac{I_1 \Delta L_1}{\Delta t} \]
\[ e_1 = -I_1 \frac{\Delta L_1}{\Delta t} - I_2 \frac{\Delta M}{\Delta t} \]
\[ e_2 = -I_2 \frac{\Delta L_2}{\Delta t} - I_1 \frac{\Delta M}{\Delta t} \]

where, \( e_1 \) = induced voltage of phase 1.
\( e_2 \) = induced voltage of phase 2
\( f_1 \) = Inductance of phase 1
\( f_2 \) = Inductance of phase 2
\( M \) = mutual inductance between the two phases.

The work done by the two power supplies during \( \Delta t \),

\[ \Delta P_i = -(e_1 I_1 + e_2 I_2) \Delta t \]

\[ = -\left[ \frac{I_1^2 \Delta L_1}{\Delta t} - I_1 I_2 \frac{\Delta M}{\Delta t} - \frac{I_2^2 \Delta L_2}{\Delta t} - I_1 I_2 \frac{\Delta M}{\Delta t} \right] \]

\[ \Delta P_i = I_1^2 \Delta L_1 + I_2^2 \Delta L_2 + 2 I_1 I_2 \Delta M \]  \( \text{(24)} \)

The increment of magnetic energy,

\[ \Delta W_m = \frac{1}{2} \Delta P_i \]

\[ = \frac{1}{2} \left[ I_1^2 \Delta L_1 + I_2^2 \Delta L_2 \right] + I_1 I_2 \Delta M \]  \( \text{(25)} \)

Mechanical output,

\[ \Delta P_o = T \Theta \]  \( \text{(26)} \)

\[ T \Theta = \frac{1}{2} \left( I_1^2 \Delta L_1 + I_2^2 \Delta L_2 \right) + I_1 I_2 \Delta M \]
Torque is expressed as,

\[ T = \frac{1}{2} I_1^2 \frac{d\phi_1}{d\theta} + \frac{1}{2} I_2^2 \frac{d\phi_2}{d\theta} + I_1 I_2 \frac{dM}{d\theta} \]

**Linear and Non Linear Analysis:**

In linear analysis, it is assumed that the magnetic material in the motor has constant permeability (i.e.,) the machine with linear magnetic characteristics means that it is having constant magnetic permeability and no magnetic saturation. The flux density is proportional to the winding current.

But in practice, variable reluctance motors do operate with their magnetic material in saturation.

---

**Diagram:**

- **Flux linkage \( \phi \)**
- **Phase current \( I \)**

Where, \( T_m \) - motor torque (N-m)

\[ J \] - inertia of motor (kg m²)
$\omega$ - angular velocity of rotor
$D$ - damping coefficient
$T_f$ - frictional load torque
$\phi_s$ - step angle (radians)
$f$ - stepping rate (steps/sec).

Frictional load torque,

$$T_f = k\omega$$

According to rotor dynamics,

$$T_m = J\frac{d\omega}{dt} + D\omega + T_f \quad (1)$$

Also, $\theta_s = \theta = \omega t = \text{step angle}$

$$\omega = \frac{\theta_s}{t} = f\theta_s \quad (2)$$

where $f = \frac{1}{t}$

Substitute (2) in (1),

$$T_m = J\frac{d}{dt}(f\theta_s) + D(f\theta_s) + T_f \quad (3)$$

where $\theta_s = \frac{360}{MN}$

(\therefore) $$T_m = J\theta_s \frac{d}{dt}(f) + D\theta_s (f) + T_f$$

If viscous friction constant is neglected, the equation will be a linear equation, the corresponding analysis is a linear analysis where linear acceleration is present.
The damping coefficient is also considered, the corresponding acceleration will be non-linear and the equation will be non-linear and the equation will be non-linear.

a) **Linear acceleration or linear analysis**:

If damping coefficient is neglected,

\[ D = 0 \]

Expression for motor torque becomes,

\[ T_m = J \frac{d\omega}{dt} + T_f \]

\[ T_m - T_f = J \frac{d\omega}{dt} \]

\[ \frac{T_m - T_f}{J} = \frac{d\omega}{dt} \]

\[ d\omega = \frac{T_m - T_f}{J} \, dt \]

On integrating,

\[ \omega = \left( \frac{T_m - T_f}{J} \right) t + \omega_i \quad \text{--- 4} \]

where \( \omega_i \) = integration constant (initial angular velocity of motor)

\( \therefore \) \( \omega = \theta_s + \omega_i \quad \text{--- 5} \)

Substitute 5 in 4,

\[ \left( \frac{T_m - T_f}{J} \right) t + \theta_s + \omega_i = \theta_s t \]
by \( \theta_s \):

\[
\left( \frac{T_m - T_f}{J_0 s} \right) t + f_1 = f
\]

\[\therefore \text{ Stepping rate}, \quad f = \left( \frac{T_m - T_f}{J_0 s} \right) t + f_1\]

\[T_f = K_0\]

Angular velocity

\[
\begin{align*}
& w_2 \\
& w_1 \\
& t_1 \\
& t_2 \quad \text{Time}
\end{align*}
\]

Linear acceleration from \( w_1 \) to \( w_2 \)

(b) Nonlinear acceleration (exponential):

\[
T_m = J_0 s \frac{df}{dt} + D_0 s f + T_f
\]

\[\left( T_m - T_f \right) = J_0 s \frac{df}{dt} + D_0 s f
\]

Dividing by \( J_0 s \)

\[
\frac{df}{dt} + \left( \frac{D}{J_0} \right) f = \frac{T_m - T_f}{J_0 s}
\]

The above equation is in the form,

\[
\frac{dy}{dx} + Py = Q, \quad \text{which have the solution of}
\]

\[
y = e^{\int P \, dx} \int Q \, e^{\int P \, dx} \, dx + c
\]
Here, \( y = f \); \( x = t \); \( p = \frac{D}{J} \) and \( a = \frac{T_m - T_f}{J B_s} = \text{constant} \)

\[
\int e^{\frac{(D/J)}{J}} \, dt = \left( \frac{\frac{T_m - T_f}{J B_s}}{J B_s} \right) \int e^{(D/J)t} \, dt + c
\]

\[
\frac{e^{(D/J)t}}{J B_s} = \left( \frac{\frac{T_m - T_f}{J B_s}}{J B_s} \right) e^{(D/J)t} + c
\]

\[
\frac{e^{(D/J)t}}{J B_s} = \left( \frac{\frac{T_m - T_f}{J B_s}}{J B_s} \right) \frac{e^{(D/J)t}}{D/J} + c \tag{6}
\]

where \( c \) is the integration constant.

To find \( c \), substitute the initial condition at \( t = 0 \).

\[
f_1 = f(0) = f_1
\]

\[
f_1 e^0 = \left( \frac{\frac{T_m - T_f}{J B_s}}{J B_s} \right) \frac{e^0}{D/J} + c
\]

\[
f_1 = \left( \frac{\frac{T_m - T_f}{J B_s}}{J B_s} \right) \left( \frac{J}{D} \right) + c
\]

\[
f_1 = \frac{T_m - T_f}{D B_s} + c
\]

\[
(\because) \quad c = f_1 - \left( \frac{\frac{T_m - T_f}{D B_s}}{D B_s} \right)
\]

\[
f e^{(D/J)t} = \left( \frac{\frac{T_m - T_f}{J B_s}}{J B_s} \right) \left( \frac{J}{D} \right) e^{(D/J)t} + \left[ f_1 - \left( \frac{\frac{T_m - T_f}{D B_s}}{D B_s} \right) \right]
\]

\[
f e^{(D/J)t} = \left( \frac{\frac{T_m - T_f}{D B_s}}{D B_s} \right) e^{(D/J)t} + \left[ f_1 - \left( \frac{\frac{T_m - T_f}{D B_s}}{D B_s} \right) \right]
\]
Divide by $e^{(D/J)t}$,

$$f = \frac{T_m - T_f}{D\theta_s} + \left[ t_1 - \left( \frac{T_m - T_f}{D\theta_s} \right) \right] e^{-(D/J)t}$$

Stepping frequency,

$$f = \frac{T_m - T_f}{D\theta_s} + \left[ t_1 - \left( \frac{T_m - T_f}{D\theta_s} \right) \right] e^{-(D/J)t}$$

This is a non-linear exponential equation which gives rise to non-linear acceleration of the rotor of the machine.

Thus, torque production is assumed as linear. In some cases, the saturation of the magnetic material of the machine leads to non-linear operation which necessitates the study of non-linear acceleration of the rotor due to the torque production.
2.11. CHARACTERISTICS OF STEPPER MOTOR

Stepper motor characteristics are divided into two groups

- Static characteristics
- Dynamic characteristics

2.11.1. Static characteristics

It is divided into two characteristics.

(i) Torque Angle curve
(ii) Torque current curve

(i) Torque-Angle curve

Torque angle curve of a step motor is shown in fig.2.19. It is seen that the Torque increases almost sinusoid ally, with angle $\Theta$ from equilibrium.

Fig. 2.19 Torque Angle

Holding Torque ($T_H$)

It is the maximum load torque which the energized stepper motor can withstand without slipping from equilibrium position. If the holding torque is exceeded, the motor suddenly slips from the present equilibrium position and goes to the static equilibrium position.

Detent torque ($T_D$):

It is the maximum load torque which the un-energized stepper motor can withstand slipping. Detent torque is due to magnetism, and is therefore available only in permanent magnet and hybrid stepper motor. It is about 5-10 % of holding torque.

Torque current curve

A typical torque curve for a stepper motor is shown in fig.2.19. It is seen the curve is initially linear but later on its slope progressively decreases as the magnetic circuit of the motor saturates.
**Torque constant (Kt)**

Torque constant of the stepper is defined as the initial slope of the torque-current (T-I) curve of the stepper motor. It is also known as torque sensitivity. Its units N-mA, kg-cm/A or OZ-in/A.

**2.11.2. Dynamic characteristics**

A stepper motor is said to be operated in synchronism when there exist strictly one to one correspondence between number of pulses applied and the number of steps through which the motor has actually moved. There are two modes of operation.

**Start-Stop mode**

Also called as pull in curve or single stepping mode.

**Slewing mode**

In start-stop mode the stepper motor always operate in synchronism and the motor can be started and stopped without using synchronism. In slewing mode the motor will be in synchronism, but it cannot be started or stopped without losing synchronism. To operate the motor in slewing mode first the motor is to be started in start stop mode and then to slewing mode. Similarly to stop the motor operating in slewing mode, first the motor is to be brought to the start stop mode and then stop.

**Start Stop mode**

Start stop mode of operation of stepper motor is shown in fig.2.21 (a). In this second pulse is given to the stepper motor only after the rotor attained a steady or rest position due to first pulse. The region of start-stop mode of operation depends on the operation depends on the torque developed and the stepping rate or stepping frequency of stepper motor.
2.12. TORQUE-SPEED CHARACTERISTICS

Torque developed by the stepper motor and stepping rate characteristics for both modes of operation are shown in fig. 2.22. The curve ABC represents the "pull in" characteristics and the curve ADE represents the "pull-out" characteristics.

The area OABCO represents the region for start-stop mode of operation. At any operating point in the region the motor can start and stop without losing synchronism. The area ABCEDA refers to the region for slewing mode of operation. At any operating point without losing synchronism to attain an operating point in the slewing mode at first the motor is to operate at a point in the start-stop mode and then stepping rate is increased to operate in slewing mode, similarly while switching off it is essential to operate the motor from slewing mode to start-stop mode before it is stopped.
Pull in torque

It is the maximum torque developed by the stepper motor for a given stepping rate in the start-stop mode of operation without losing synchronism. LM represents the pull in torque (i.e.) TPI corresponding to the stepping rate F (i.e.) OL.

Pull out torque

It is the maximum torque developed by the stepper motor for a given stepping rate in the slewing mode without losing synchronism. LN represents the pull in torque (i.e.) TPO corresponding to F (i.e.) OL.

Pull in range

It is the maximum stepping rate at which the stepper motor can operate in start-stop mode developing a specific torque (without losing synchronism). PIT represents pull in range for a torque of T (i.e.) OP. This range is also known as response range of stepping rate for the given torque T.

Pull out range

It is the maximum stepping rate at which the stepper motor can operate in slewing mode developing a specified torque without losing synchronism. PIPO represents the pull out range for a torque of T. The range PIPO is known slewing range.

Pull in rate (FPI)

It is the maximum stepping rate at which the stepper motor will start or stop without losing synchronism against a given load torque T.

Pull out rate (FPO)

It is the maximum stepping rate at which the stepper motor will slew, without missing steps, against load torque T.

Synchronism

This term means one to one correspondence between the number of pulses applied to the stepper motor and the number of steps through which the motor has actually moved.

Mid frequency resonance

The phenomenon at which the motor torque drops to a low value at certain input pulse frequencies.
2.13 DRIVE SYSTEM AND CONTROL CIRCUITRY FOR STEPPER MOTOR

2.13.1 DRIVE SYSTEM

The stepper motor is a digital device that needs binary (digital) signals for its operation. Depending on the stator construction, two or more phases have to be sequentially switched using a master clock pulse input. The clock frequency determines the stepping rate, and hence the speed of the motor. The control circuit generating the sequence is called a translator or logic sequencer.

![Fig. 2.23 Block Diagram of the drive system of a stepping motor.](image)

The fig 2.23 shows the block diagram of a typical control circuit for a stepper motor. It consists of a logic sequencer, power driver and essential protective circuits for current and voltage limiting. This control circuit enables the stepper motor to be run at a desired speed in either direction. The power driver is essentially a current amplifier, since the sequence generator can supply only logic but not any power. The controller structure for VR or hybrid types of stepper motor.

![Fig. 2.24 Block diagram of a typical step motor control](image)

2.13.2 LOGIC SEQUENCER

The logic sequencer is a logic circuit which control the excitation of the winding sequentially, responding to step command pulses. A logic sequencer is usually composed of a shifter register and logic gates such as NANDs, NORs etc. But one can assemble a logic sequencer for a particular purpose by a proper combination of JK flip flop, IC chips and logic gate chips.

Two simple types of sequencer build with only two JK-FFs are shown in fig 2.39 for unidirectional case. Truth tables for logic sequencer also given for both the directions.
TABLE 2.7 Logic Sequencer

<table>
<thead>
<tr>
<th>R</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ph A,Q1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Ph B,Q2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Ph A,Q1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ph B,Q2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 2.25 A unidirectional logic sequencer for two phase operation of a two phase hybrid motor

The correspondence between the output terminals of the sequencer and the phase windings to be controlled is as follows.
If Q1 is on the H level the winding Ph A is excited and if Q1 is on L level, Ph A is not excited.

To reserve the rotational direction, the connection of the sequencer must be interchanged. The direction switching circuits shown in fig 2.40 may be used for this purpose. The essential functions being in the combination of three NAND gates or two AND gates and a NOR gate.

2.13.3. Power Driver Circuit

The number of logic signals discussed above is equal to the number of phases and the power circuitry is identical for all phases. Fig. 2.26(a) shows the simplest possible circuit of one phase consisting of a Darlington pair current amplifier and associated protection circuits. The switching waveform shown in fig. 2.26(c) is the typical R-L response with an exponential rise followed by decay at the end of the pulses.

In view of the inductive switching operation, certain protective elements are introduced in the driver circuit. These are the inverter gate 7408, the forward biased diode D1 and the freewheeling diode D. The inverter IC provides some sort of isolation between the logic circuit and the power driver.

There are some problems with this simple power circuit. They can be understood by considering each phase winding as a R-L circuit shown in fig. 2.26(b) subject to repetitive switching. On application of a positive step voltage, the current rises exponentially as

\[ i(t) = I(1 - e^{-\frac{t}{\tau}}) \]  

Where \( I = V/R \) – rated current and \( \tau = L/R \) winding time constant.

Fig. 2.26 Power Driver Stage of Stepper Motor Controller
In practice, the time constant $\tau$ limits the rise and fall of current in the winding. At low stepping rate the current rises to the rated value in each ON interval and falls to zero value in each OFF interval. However as the switching rate increases, the current is not able to rise to the steady state, nor fall down to zero value within the on/off time intervals set by the pulse waveform. This in effect, smoothens the winding current reducing the swing as shown in fig. 2.45. As a result the torque developed by the motor gets reduced considerably and for higher frequencies, the motor just vibrates or oscillates within one step of the current mechanical position.

![Fig. 2.27 Effect of increasing Stepping Rate on Current Swing](image)

In order to overcome these problems and to make improvement of current build up several methods of drive circuits have been developed.

For example when a transistor is turned on to excite a phase, the power supply must overcome effect of winding inductances has tendency to oppose the current built up. As switching frequency increases the position that the buildup time takes up within the switching cycle becomes large and it results in decreased torque and slow response.

### 2.13.4 Improvement of current buildup/special driver circuit

**(a) Resistance drive (L/R drive)**

Here the initial slope of the current waveform is made higher by adding external resistance in each winding and applying a higher voltage proportionally. While this increases the rate of rise of the current, the maximum value remains unchanged as shown in fig.

![Fig. 2.28 L/R drive](image)
The circuit time constant is now reduced and the motor is able to develop normal torque even at high frequencies. The disadvantage of this method is

Flow of current through external resistance causes $I^2R$ losses and heating. This denotes wastage of power as far as the motor is concerned.

In order to reach the same steady state current $I_R$ as before, the voltage required

To be applied is much higher than before. Hence this approach is suitable for small instrument stepper motor with current ratings around 100 mA, and heating is not a major problem.

\(b\) Dual voltage driver (or) Bi-level driver

To reduce the power dissipation in the driver and increase the performance of a stepping motor, a dual-voltage driver is used. The scheme for one phase is shown in fig. 2.29.

When a step command pulse is given to the sequencer, a high level signal will be put out from one of the output terminals to excite a phase winding. On this signal both $T_1$ and $T_2$ are turned on, and the higher voltage $E_H$ will be applied to the winding. The diode $D_1$ is now reverse biased to isolate the lower voltage supply. The current build up quickly due to the higher voltage $E_H$. The time constant of the monostable multivibrator is selected so that transistor $T_1$ is turned off when the winding current exceeds the rated current by a little. After the higher

![Fig. 2.29 Improvement of current buildup by dual voltage drive](image)

Voltage source is cut off the diode is forward biased and the winding current is supplied from the lower voltage supply. A typical current wave form is shown in fig.
Fig. 2.30 Voltage and current wave form in dual voltage driver

When the dual voltage method is employed for the two phase on drive of a two phase hybrid motor, the circuit scheme will be such as that shown in fig. Two transistor $T_1$ and $T_2$ and two diodes $D_1$ and $D_2$ are used for switching the higher voltage. In dual voltage scheme as the stepping rate is increased, the high voltage is turned on for a greater percentage of time.

Fig. 2.31 A dual-voltage driver for the two-phase-on drive of a two phase hybrid motor

This drive is good and energy efficient. However it is more complex as it requires two regulated power supplies EH& EL and two power transistor switches Tr1 & Tr2 and complex switching logic. Hence it is not very popular.

(c) Chopper drive

Here a higher voltage 5 to 10 times the related value is applied to the phase winding as shown in fig. and the current is allowed to raise very fast. As soon as the current reaches about 2 to 5% above the rated current, the voltage is cut off, allowing the current to decrease exponentially. Again as the current reaches some 2 to 5% below the rated value, the voltage is applied again. The process is repeated some 5-6 times within the ON period before the phase is switched off. During this period the current oscillates about the rated value as shown in fig. A minor modification is to chop the applied dc voltage at a high frequency of around 1khz, with the desired duty cycle so as to obtain the average on-state current equal to the rated value.
The chopper drive is particularly suitable for high torque stepper motors. It is energy efficient like the bi-level drive but the control circuit is simpler.

**(d) Problems with driver circuits**

A winding on a stepping motor is inductive and appears as a combination of inductance and resistance in series. In addition, as a motor revolves a counter emf is produced in the winding. The equivalent circuit to a winding is hence, such as that shown for designing a power driver one must take into account necessary factors and behavior of this kind of circuit. Firstly the worst case conditions of the stepping motor, power transistors, and supply voltage must be considered. The motor parameters vary due to manufacturing tolerance and operating conditions. Since stepping motors are designed to deliver the highest power from the smallest size, the case temperature can be as high as about 100°C and the winding resistance therefore increases by 20 to 25 per cent.

**Suppressor circuits**

These circuits are needed to ensure fast decay of current through the winding when it is turned off. When the transistor in the above fig is turned off a high voltage builds up to \( L\frac{di}{dt} \) and this voltage may damage the transistor. There are several methods of suppressing this spike voltage and protecting the transistor as shown in the following.

**(a) Diode suppressor**

If a diode is put in parallel with the winding in the polarity as shown in fig. a circulating current will flow after the transistor is turned off, and the current will decay with time. In this scheme, no big change in current appears at turn off, and the collector potential is the supply potential \( E \) plus the forward potential of the diode. This method is very simple but a drawback is that the circulating current lasts for a considerable length of time and it produces a braking torque.

![Fig. 2.33 Diode suppressor](image-url)
(b) Diode-Resistor suppressor

A resistor is connected in series with the diode as shown in fig to damp quickly the circulating current. The voltage VCE applied to the collector at turn-off in this scheme is

\[ V_{CE} = E + I R_s + V_D \]

Where
- \( E \) = supply potential
- \( I \) = Current before turning off
- \( R_s \) = resistance of suppressor resistor
- \( V_D \) = forward potential of diode

![Diode-Resistor suppressor diagram](image)

Fig. 2.34 Diode-resistor suppressor

A high resistance \( R_s \) is required to achieve a quick current decay, but this also results in a higher collector potential \( V_{CE} \), thus a transistor with a high maximum voltage rating is necessary.

(a) Zener diode suppressor

In this zener diode, are often used to connect in series with the ordinary diode as shown in fig. Compared with preceding two cases zener diode which provides negative bias causes the current to decay more quickly after turn off. In addition to this, it is a merit of this method that the potential applied to the collector is the supply potential plus the zener potential, independent of the current. This makes the determination of the rating of the maximum collector potential easy. However, zeners are signal diodes, rather than power diodes. Their power dissipation is limited to 5w. Consequently, this suppressor can be used for very small instrument stepper motors of typical size 8 to 11.

Comparison of effects of various suppressor schemes of various suppressor schemes
d) Condenser suppressor

This scheme is often employed for bifilar-wound hybrid motor. An explanation is given for the circuit shown in fig:

Fig. 2.37 Condenser suppressor

A condenser is put between ph A and ph A⊥ and between ph B and ph B⊥. These condensers serve two fold purposes.

1. When a transistor is turned off, the condenser connected to it via a diode absorbs the decaying current from the winding to protect the transistor.

Let us see the situation just after the Tr 1 is turned off in the one phase on mode. Either Tr 2 or Tr 4 will turn on, but Tr 3 will still be in the turned off state. Since the winding of ph A & ph A⊥ are wound in the bifilar fashion, a transient current will circulate in loop. If Tr 3 is turned on when the transient current becomes zero and the charge stored in the condenser becomes maximum, a positive current can easily flow through phase A winding. By this resonance mechanism, currents are used efficiently in this scheme. This feature remains in the two phase on mode too. The condenser suppressor is suited to drives in which stepping rate is limited in a narrow range.

2. Another utility of condensers is as an electrical damper, a method of damping rotor oscillations is to provide a mechanism to convert kinetic energy to joule heating. If a rotor
having a permanent magnet oscillates, an alternating emf is generated in the winding. However if a current path is not provided or a high resistance is connected, no current will be caused by this emf. When the condenser is connected between phases an oscillatory current will flow in the closed loop and joule heat is generated in the windings, which means that the condenser works as an electrical damper.

2.14 APPLICATION OF STEPPER MOTOR:

The main application of stepper motor may be divided into the following groups.

1. Instrumentation applications.
2. Computer peripherals & Office equipment’s.
3. Numerical control of machine tools and robotics.
4. Applications in semiconductor technology.
5. Space vehicles and satellites.
6. Electro medical and
7. Miscellaneous applications.

1. Instrumentation application:

This involve low torque applications such as

Quartz watches.
Synchronized clocks.
Camera shutter operations.

2. Stepper motor application in computer peripherals:

This involve medium torque, high performance and high volume application such as

Dot matrix and line printers.
Graph plotters.
Floppy disk drives
Digital X-Y plotters.
Magnetic tape drives.
Paper tape drives.
3. Application is office equipment:

Electronic typewriters.
Copiers
Facsimile machines.

4. Machine tool applications:

This involve high torque application such as
Numerical control system for milling machine
X-Y tables and index table.
Home use and industrial sewing machines.

5. Application in semiconductor technology:

Stepper motors used in high vacuum.
Goniometer-An instrument used to determine crystalline structure.
Electron beam micro fabricator.

6. Stepper motor used in space vehicles and satellites.

7. Robotics.

8. Electro medical applications:

This involve high torque applications such as
X-ray machines.
Radiation therapy units.
Ultra sound scanner.

9. Miscellaneous applications:

Nuclear reactors.
Heavy industry applications.
Automatic focusing mechanism in camera